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THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

LXXVII

AUGUST 1902

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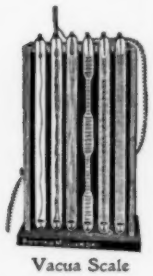

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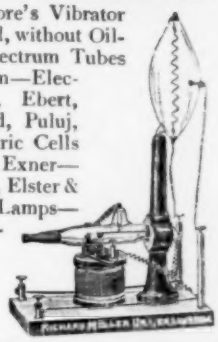

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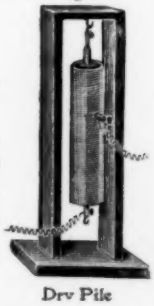

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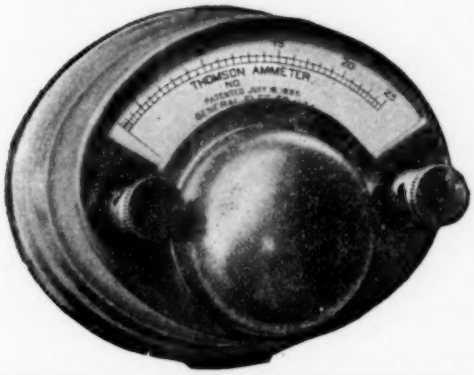

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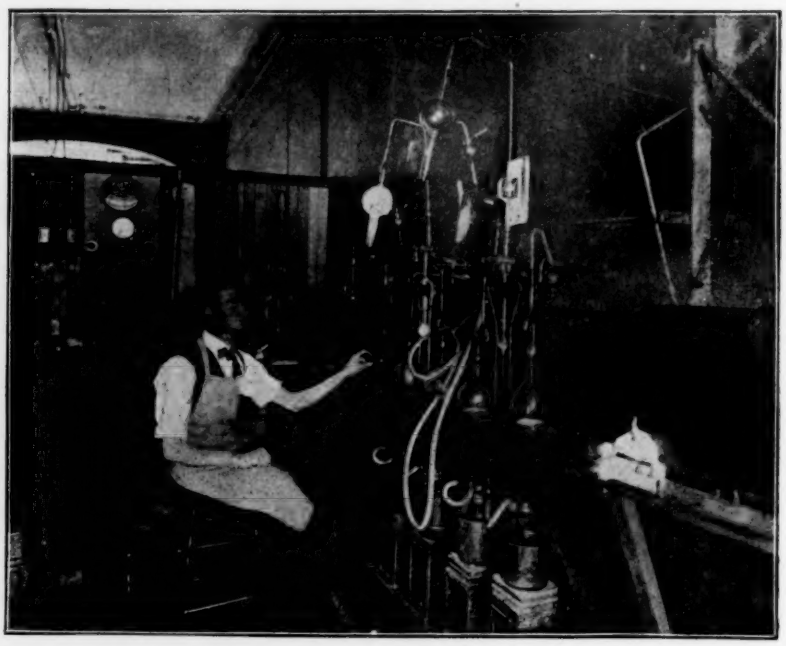

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THE

PHYSICAL REVIEW.

ON THE SPECIFIC HEAT OF SUPERCOOLED WATER.

By H. T. Barnes and H. Lester Cooke.

THE determination of the variation of the specific heat of water in the neighborhood of the freezing point is a matter of interest on account of the rapid increase in specific heat as the temperature decreases. This has been conclusively shown by Rowland,[1] Griffiths,[2] Lüden,[3] and one of the authors [4] by widely differing methods. The question was first raised by Rowland in his memoir as to whether this rapid increase meant that the specific heat became infinitely great at the freezing point. Since it is comparatively easy by taking proper precautions to supercool water the idea of an infinitely large specific heat is untenable. The idea has also been advanced that the maximum of specific heat must occur at the point of maximum density. This also seems unlikely on account of the fact that the curve representing the variation of the specific heat between 0° and 100° C. does not resemble the curve of density, and is apparently independent of it.

In the present paper the authors describe some experiments recently carried out to determine if possible any deviation from the variation curve below the freezing point. Since supercooling is in every way a continuous process it is unlikely that any other relation can hold than that represented by a continuation of the varia-

[1] Proc. Amer. Acad., 15, 75 (1879).
[2] Phil. Trans, 184, 361 (1893).
[3] Beiblätter, 1897.
[4] Proc. Roy. Soc., 67, 238 (1900).

tion curve in this region. No indication of a change in the shape of the curve as 4° C. is reached was noticed in the measurements made previously by one of the authors. Absolute measurements of the thermal capacity in terms of the electrical units were made with great accuracy at 4° C., others of less accuracy were made at intervals down to half a degree above zero. Had the specific heat passed through a maximum at 4° C. some indication of the change would have been apparent before this point was reached. No indication whatever could be observed, and further, the change in the slope of the curve instead of decreasing increased more rapidly in this region. The equation representing the variation accurately between 10° and 50° C. yields results therefore somewhat below the observed values.

In selecting a method of measurement we were somewhat restricted on account of the instability of water below zero. We made some preliminary experiments with a view of finding out if water free from dissolved air and dust particles could be supercooled while flowing through a tube. We hoped at the outset to be able to carry out our experiments by a continuous method of mixture. The water was prepared by condensing steam under oil in an air-free space, and was then led through a spiral of pure tin tubing immersed in a freezing mixture. We were unable however to maintain a flow on account of the formation of ice. Had we started initially with supercooled water it is probable we should have been more successful since our later experiments seem to indicate that it is more difficult to pass the freezing point in supercooling without ice forming, than it is to lower the temperature once the mass has become undercooled.

Our next method was one in which the supercooled water was frozen by sudden agitation and the amount of ice determined. With different degrees of supercooling the quantity of ice formed in bringing the temperature of the water back to the freezing point would give a measure of the thermal capacity. We had to leave this method, however, for the time being, on account of the difficulty of estimating the quantity of ice accurately. We finally selected a method of mixture which, although not yielding us results of very great accuracy, yet gave us measurements sufficient to show the

kind of variation. Our method was very simple and independent of any large or uncertain corrections.

A calorimeter, made of polished copper, was suspended in a polished metal vessel, which in turn was surrounded by heavy lagging. The temperature of the air in this vessel, as well as the temperature of the mixture, was obtained on a Müller mercury thermometer graduated to a fiftieth of a degree. Two flasks of equal capacity were provided, and into one was placed about 200 grams of carefully prepared dust-free water, and into the other an equal weight of ordinary distilled water. Two baths were provided, one for the low temperatures, and the other for a corresponding high temperature. On cooling the dust-free water to the desired temperature the other flask was placed in the hot bath and the temperature raised until it was the same amount above the temperature of the calorimeter as the cool water was below. It was then maintained constant at this point. On quickly removing the flasks from the two baths at the same time and pouring equally into the calorimeter the resulting temperature of the mixture was observed. The temperature of the mixture seldom differed much from the temperature of the calorimeter, so that only an approximate correction for the heat of the calorimeter, stirrer and thermometer bulb had to be applied. The accuracy of the method depended on :

(*a*) The accuracy of the thermometers used.

(*b*) The fact that the hot water would cool in transferring the contents of the flasks to the calorimeter as much as the cold water would become warmed.

(*c*) That the calorimeter never became heated or cooled much in the mixing, which required that the water from the two flasks be poured-in at the same time and immediately mixed by a suitable stirrer.

For obtaining the temperature of the cold water a second Müller thermometer graduated to a fiftieth of a degree was used, which had a scale reading from 15° C. below zero to 2° above. The hot flask was read by means of a Müller thermometer graduated to a tenth of a degree. We did not happen to have available a thermometer more finely graduated and reading at the point we required, but with a telescope we could read the temperature on this

thermometer to a hundredth of a degree with comparative certainty. Our mixture thermometer had a scale from 0° to 2° and from 15° to 32°, a bulb in the capillary tube allowing for the interval between 15° and 2°. For calibrating these thermometers we had the zero mark in all three. This was all that was required in the low temperature thermometer, since the readings were only required at zero and a few degrees below. For the other thermometer we had a Geissler thermometer graduated to tenths, which had been originally calibrated all over its scale from 0° to 50° by comparison with a platinum thermometer, and which had been recently recompared at 16° as a check. By comparing our two thermometers with this directly the readings could be reduced to the nitrogen scale. This comparison showed that our mixture thermometer read 0.270 of a degree too high at 20°, which was just about the temperature of the mixture, and 0.320° too high at 0°. Our hot thermometer read 0.04 of a degree too high at 40° and 0.07 of a degree too high at 0°.

As a check of the accuracy of the method our first experiments were made to determine the specific heat of water between 0° and 20° since from the variation curve we could obtain the mean specific heat over this range with great accuracy. We also obtained the mean specific heat over the higher range for the warm water from the same variation curve. The results of these and subsequent experiments were expressed in terms of the thermal unit at 16°, which one of the authors has shown is nearly equal to the mean over the entire range between 0° and 100°. Griffith's unit, which was recommended at the Paris congress in 1900[1] for adoption, is practically equal to this, and is defined as the heat required to raise the temperature of one gram of water from 15° to 16°.

The lowest temperature we obtained was — 5.55° in our experiments. We found it somewhat difficult to cool water below — 6° since the water has to be stirred, and is required to pour freely from the flask. No great difficulty was experienced down to this temperature in either respect. In the process of supercooling the water was always cooled first to 0° in a freezing-point mixture, and then transferred to successive mixtures of salt and snow arranged

[1] Reports of the Physical Congress at Paris, Vol. 1, p. 226 (1900).

in steps down to the point required. On several occasions ice formed around the walls of the flask on transferring to the first cold bath, but this never happened once the mass of water had started to cool below zero. When freezing occurred at the low temperatures, as it did in one experiment when we tried to reach — 10°, the ice formed all through the mass of the water. Care had to be taken that the thermometer bulb never touched the bottom of the flask when the water was supercooled, as it almost invariably caused freezing to take place. The ice would start at the point of contact and immediately spread out all through the mass of the water.

In the following table will be found the details of the observations. Under the second column is given the weight in grams and the temperature of the cold water before mixture. The third column contains similar data for the warm water. Under calorimeter temperature is given the temperature indicated by the mixture thermometer before the water was poured in. The last column contains the resulting temperature of the mixture. The temperatures given are all corrected for the errors in the various thermometers.

TABLE OF OBSERVATIONS.

No. of Experiment.	Cold Water.		Hot Water.		Calorimeter, Temp.	Mixture, Temp.
	Weight.	Temp.	Weight.	Temp.		
1	190.54	0.005°	191.29	39.52°	19.00°	19.710°
2	213.97	0.007	213.74	39.94	19.27	19.910
3	219.69	0.030	220.05	38.86	20.80	19.470
4	212.83	−3.310	213.02	43.38	17.96	19.370
5	211.91	−3.910	213.10	43.63	19.71	19.870
6	244.17	−4.300	244.54	39.73	17.63	17.700
7	216.82	−5.550	216.98	44.04	21.13	19.250

The water equivalent of the calorimeter, thermometer bulb and stirrer was calculated to be 17.65 grams.

In the next table we give the result of the experiments. The first column gives the range through which the cold water was warmed. The second the mean specific heat over this range calculated from the observations, and in the third the mean specific heat obtained from the variation curve for the absolute determina-

tions of the thermal capacity of water between the freezing and boiling points.

The thermal capacity of the warm water was always less than one, this value between 20° and 40° being 0.9977 in terms of the 16-degree unit.

SPECIFIC HEAT OF SUPERCOOLED WATER.

Method of Mixture.

No. of Experiment.	Range.	Mean Specific Heat Observed.	Mean Specific Heat From Curve.
1	0 to 19.7°	1.0037	1.0030
2	0 to 19.9	1.0000	1.0030
3	0 to 19.5	1.0024	1.0031
4	−3.31 to 19.9	1.0062	1.0043
5	−3.91 to 19.9	1.0017	1.0044
6	−4.30 to 17.7	1.0007	1.0050
7	−5.55 to 19.2	1.0059	1.0053

This gives, taking the average of the three determinations from 0° to 20°, 1.0020 for the specific heat while the variation curve gives 1.0030. The mean of the observations for − 3.31° and − 3.91° gives 1.0040, while the curve gives 1.0044. The observation at − 4.30 shows a discrepancy of 4 parts in 1,000, which is more likely to be ascribed to the uncertainty of the method than to an indication of a lower specific heat below 0°. The observation at − 5.55°, which is the lowest we were able to obtain, agrees very closely.

The following observations are taken from Rowland's experiments by the method of mixtures, as quoted by E. H. Griffiths in his memoir :[1]

$$\frac{0° \text{ to } 18°}{18° \text{ to } 27°} = 1.0025, \quad \frac{0° \text{ to } 17°}{17° \text{ to } 29°} = 1.0024,$$

$$\frac{0° \text{ to } 18°}{18° \text{ to } 30°} = 1.0067, \quad \frac{0° \text{ to } 16°}{16° \text{ to } 24°} = 1.0010,$$

$$\frac{0° \text{ to } 17°}{17° \text{ to } 25°} = 1.0027, \quad \frac{0° \text{ to } 21°}{21° \text{ to } 28°} = 1.0045.$$

[1] L. c., p. 494.

These values, which are the best of his determinations, are a little higher than ours on account of their being expressed in terms of the specific heat of the warm water and not in terms of a unit at 16°. The order of accuracy of Rowland's determinations is about the same as in our own, and shows we think just about what can be expected from a method of mixture carried out with a tolerable amount of accuracy.

We may assume then from the result of these experiments that the variation curve for the specifie heat of water extends below zero for supercooled water, which is a result that might have been expected, but which it is interesting to have shown experimentally.

The following table contains the values of the specific heat of water in terms of the 16-degree unit between −5° and 100° C.:

VARIATION OF THE SPECIFIC HEAT OF WATER IN TERMS OF A THERMAL UNIT AT 16° C.

Temperature.	Spec. Heat.	Temperature.	Spec. Heat.
−5°C.	1.0158	50	0.99800
0	1.0094	55	0.99850
+ 5	1.00530	60	0.99910
10	1.00230	65	0.99970
15	1.00030	70	1.00035
20	0.99895	75	1.00100
25	0.99806	80	1.00166
30	0.99759	85	1.00237
35	0.99735	90	1.00305
40	0.99735	95	1.00370
45	0.99760		

The absolute value of the thermal capacity of water at zero may be taken to be 4.228 joules in terms of the electrical units, assuming for the Clark cell the value 1.4342 int. volts at 15° C. and the international ohm as correct. In terms of the value of the Clark cell 1.43325 int. volts at 15° C., which one of the authors[1] has shown brings the electrical determinations into agreement with the mechanically measured values of Rowland and of Reynolds and Moorby, the value comes 4.222 joules.

[1] L. c., p. 244.

This latter value is probably correct since it is based on a value of the Clark cell which agrees so nearly with the absolute measurements made by Professor Carhart at Ann Arbor, *i. e.*, 1.4333 int. volts.

McDONALD PHYSICAL LABORATORY,
McGILL UNIVERSITY, March 31, 1902.

THE PENETRATION OF TOTALLY REFLECTED LIGHT INTO THE RARER MEDIUM.

By Elmer E. Hall.

THE fact that total reflection does not take place at the geometrical boundary of two media was discovered and experimentally investigated by Newton.[1] According to Newton the path of the ray during total reflection was a parabola,[2] the vertex being within the rarer medium. The problem was both experimentally and theoretically studied by Fresnel,[3] and more or less treated by Verdet,[4] Young,[5] Huygens,[6] Biot,[7] Babinet,[8] Billet,[9] Stokes,[10] and others. The first reliable quantitative work was published by G. Quincke[11] in 1866.

The purpose of the present paper is to investigate quantitatively the penetration in thin films, from the first boundary of which "total reflection" takes place, and to extend the study qualitatively to the case in which the second or rarer medium is no longer thin.

Introduction.

Many of the phenomena in the case of thin films may be easily obtained qualitatively. If two glass prisms be pressed together, and if the thickness of the air film between them be sufficiently small, total reflection no longer takes place in the air-layer between

[1] Newton, Opticks, Book II., Obs. 1, 2, 8; Book III., qu. 29.

[2] Newton, Principia, Phil. Nat., Book I., Prop. 96.

[3] Ann. d. Chem. et Phys., t. 23, 1823, p. 130; t. 29, 1825, p. 175; t. 46, 1831, p. 225; also Fresnel, Oeuvres, t. I., pp. 447, 453, 767.

[4] Verdet, Oeuvres, t. VI., pp. 430 and 563.

[5] Young's Lectures on Nat. Phil., 1807, I., p. 461, II., p. 623; also Phil. Trans., 12, 1801.

[6] Huygens, Traité da la Lumiere, 1690, Leide, p. 38.

[7] Biot, Traité de Physique, 1816, t. III., pp. 276, 290.

[8] Comptes Rendus, t. 8, 1839, p. 709.

[9] Ann. de Chemie, t. 64, 1862, p. 410.

[10] Phil. Trans., 8, 1849, p. 642.

[11] Pogg. Ann., Band 127, 1866, pp. 1 and 199.

the two prisms, and light is transmitted at all angles of incidence. That the two prisms are not in actual contact may be seen from the interference bands obtained when the angle of incidence is less than the critical angle. If a sunbeam, whose intensity has been reduced by a ground-glass screen, fall upon the air layer at an angle greater than the critical angle a portion of the light will be transmitted if the prisms be sufficiently close together. If the back prism now be pulled slowly away until transmission no longer occurs, and then the ground-glass screen be removed, allowing the full intensity of the sunlight to fall upon the prisms, transmission of a portion of the light again occurs, showing that an increase in intensity increases the depth to which the vibrations in the air layer extend. On separating the prisms the light becomes red before vanishing, showing that the longer wave-lengths affect the air layer to the greater depth.

The phenomena are best seen when one of the prisms has a convex surface. This gives an elliptical spot of transmitted light whose major diameter changes with the angle of incidence, the intensity, the wave-length, and, if the incident light be plane polarized, with the plane of polarization. In fact, the more important conclusions of Quincke can be verified qualitatively without difficulty. These conclusions may be summarized as follows.

1. The diameter of the transmitted elliptical spot of light, and therefore the penetration, increases with an increase in the intensity of the incident light.

2. In the neighborhood of the critical angle the color bands for a white light source are lost in dark bands. The diameter of the transmitted spot increases rapidly with increasing angle of incidence, reaching a maximum just as the dark bands disappear. The penetration then at first rapidly and later more slowly decreases with increased angle of incidence.

3. The penetration increases with the wave-length of the incident light.

4. As the angle of incidence is increased from the critical angle the penetration is at first greater for light polarized perpendicular to the plane of incidence, and then greater for light polarized parallel to the plane of incidence.

5. The penetration increases for an increase in the index of refraction of the film.

6. If the incident light be plane polarized the light which penetrates the film and passes through the second prism as well as that which is "totally reflected" is elliptically polarized.

Before obtaining a prism with the proper convex surface the writer employed the following method to obtain quantitative results. A thin layer of silver was deposited on one edge of the plane hypotenuse surface of a rectangular prism. This surface was placed in contact with the hypotenuse surface of a similar prism and the two prisms were placed in a brass clamp with edges vertical. The clamp was provided with set screws at top and bottom so that adjustment could be made to give interference bands parallel to the edge of the prism and extending from the silver nearly to the other side of the prism. The bands disappeared for an angle of incidence equal to or greater than the critical angle, and light was transmitted where before were interference bands. The nearer the part through which transmission occurred approached the silvered edge the greater was the penetration. The distance between the prisms at any point could be obtained from the interference bands. The method gives good qualitative results, but is not so well adapted for quantitative work as the method using a convex surface.

Apparatus and Method.

Two rectangular prisms, one of flint and one of crown glass, were ground by John A. Brashear for the University of California, where the writer began the investigation. The side face were square, being 25 mm. on each edge, and approximately plane. The hypotenuse surfaces were convex, being ground accurately spherical with a radius of curvature of about eight meters. These prisms were loaned the writer by the Physics Department of the University of California and the investigations continued at Cornell University.

A 90° prism with a good plane hypotenuse surface was used with one of the prisms having the hypotenuse surface slightly convex. The two prisms with their hypotenuse surfaces together were placed in a brass clamp. The clamp was provided with two

set screws so that the pressure and position of contact could be regulated easily. The clamp containing the prisms P (see Fig. 1), was placed on the table of a spectrometer provided with leveling screws so that the prism edges could be set vertically.

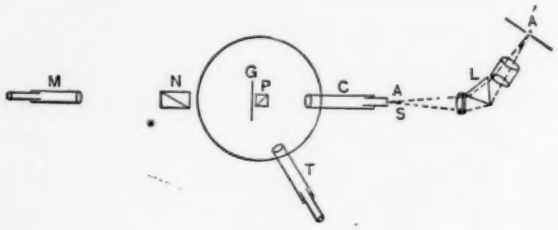


Fig. 1.

The light was incident on the prism whose hypotenuse surface was plane. From two to three millimeters back of the prisms was a ground-glass screen G, and back of this a Nicol prism N. The vertical diameter of the transmitted spot of light was measured by means of a telescope M, provided with a micrometer eyepiece. The values of the micrometer readings were obtained by replacing the ground-glass screen by a glass scale ruled to half millimeters. The magnification was such that one complete revolution of the micrometer screw corresponded to a distance of a little less than half a millimeter. The collimator C was focused for parallel rays. The focal length of the collimator lens was 34 cm., and the slit width in most of the work was 0.9 mm. The source was a single-jet acetylene burner placed at A for a white light source and at A' for an approximately monochromatic source, L being a lens and prism system to throw a pure spectrum on the slit. The spectrometer circle was graduated to 5 minutes of arc with a vernier reading to 5 seconds. Normal incidence was obtained by the usual spectrometer method, the cross hairs of the telescope T being set on a silk fiber stretched vertically over the center of the slit. The angle of incidence on the prism face could then be determined to within a few seconds of arc.

The thickness of the layer between the prisms was obtained by measuring the diameter of the interference rings for several angles of incidence less than the critical angle. The source for this pur-

pose was a sodium flame s placed in front of the slit. The thickness d of the layer could be calculated from the formula

$$d = \frac{n\lambda}{4 \cos r}$$

where r is the angle of refraction and λ the wave-length within the film, and n takes the values 1, 3, 5, etc., for dark rings in the transmitted system. A calibration curve was plotted with diameter of rings as ordinates and thickness of film as abscissas. From this curve the thickness of the film penetrated could be obtained from the diameter of the transmitted spot of light. An air film was generally used in the calibration. The prisms were put together with enough pressure to make them in actual contact in the central part of the hypotenuse face. This was necessary since the introduction of a liquid film by placing a drop of liquid on the top of the prisms has a tendency to pull the prisms closer together. A new curve was plotted each time the prisms were put together.

The angle between the prism face on which the light was incident and the hypotenuse face was 45° 2′ 32″ for the crown-glass prism, and 44° 56′ 20″ for the flint-glass prism. The index of refraction, for sodium light, of the crown-glass prism having a convex hypotenuse surface was 1.5164, and for the similar flint-glass prism the index was 1.6166.

In working with films whose index of refraction differed little from the glass, as in the case of benzole and crown glass, an arrangement of the prisms as shown in Fig. 2 was used. Light was incident on prism A. B was the prism with a convex surface, C was stuck to B with Canada balsam to prevent total reflection at the side surface.

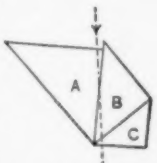


Fig. 2.

Great care was taken to have the glass surfaces clean. They were washed in nitric acid, in a solution of caustic potash in alcohol, and in distilled water. Water marks, when present, were removed by gently rubbing the surfaces with the optical paper furnished by Bausch and Lomb. Dust particles were removed with a camel's-hair brush. The room was kept partially darkened and the eye was rested every few minutes throughout a set of measurements.

OBSERVATIONS WITH WHITE LIGHT.

Using a white light source the results for a number of substances are plotted in Fig. 3. Angles of incidence on the film are plotted as abscissas, and the thicknesses of the film, through which light was transmitted to the second prism, expressed in thousandths of

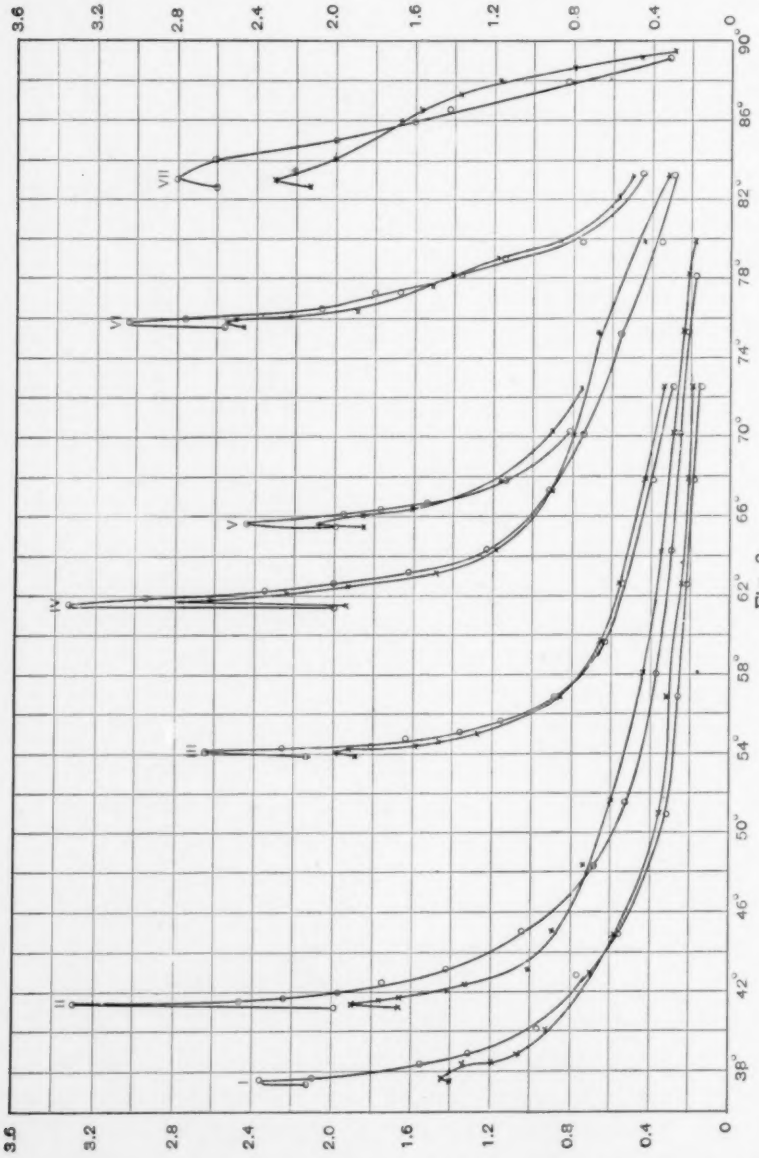


Fig. 3.

I., Flint-glass, air, flint-glass; II., Crown-glass, air, crown-glass; III., Flint-glass, water, flint-glass; IV., Crown-glass, water, crown-glass; V., Flint-glass, benzole, flint-glass; VI., Crown-glass, turpentine, crown-glass; VII., Crown-glass, benzole, crown-glass.

millimeters are plotted as ordinates. The curves have the same form for all substances used, and if produced they would intersect at the point whose coördinates are 90° and 0°. When the light was polarized perpendicular to the plane of incidence the observed points are indicated by small circles, and when the light was polarized in the plane of incidence the observed points are indicated by small crosses. In the case of such substances as air, water, and benzole the index of refraction obtained with a hollow prism agreed with the value obtained from the angle of incidence for which the bands disappeared and the diameter of the transmitted spot of light was a maximum. In the case of benzine and turpentine such was not the case. For turpentine the index of refraction by the prism method was 1.4671, and by calculation from the incidence angle for which sodium bands disappeared the index was 1.4690, the result varying slightly for different films. This disagreement may be due to the fact that turpentine is not a single chemical compound, and because of greater surface tension, or greater adhesion for the glass, that portion having the higher index of refraction is drawn between the prisms to form the film.

For angles of incidence equal to the critical angle the edges of the transmitted spot of light become diffuse, and it becomes difficult to determine just where the edge is. In most of the curves in Fig. 3 the maximum penetration given corresponds to angles of incidence a few minutes greater than the critical angle. That portion of each curve before the maximum is reached gives the diameter of the transmitted spot within the first dark ring.

Monochromatic Source.

In order to get measurements which might be compared with theory an approximately monochromatic source of fairly constant intensity was necessary. This was secured by allowing the desired portion of the spectrum of the acetylene source to fall upon the collimator slit. The lenses used were achromatic. The one between the prism and the collimator was of the same focal length as the collimator lens, thus minimizing the error due to a wide slit.

In the following tables each result is the average of from 10 to 30

readings, not differing widely among themselves. The latter i gives the angle of incidence on the face of the first prism, φ the angle of incidence on the film, d and d/λ give, respectively, the penetrations in the film expressed in thousandths of millimeters and in wave-lengths within the film of the light used. The symbols $\perp$ and // denote that the plane of polarization is perpendicular and parallel respectively to the plane of incidence. (Fresnel interpretation.) μ_1 is the index of refraction of the first prism, μ_2 of the film, and μ the relative index between the film and the first prism. The critical angle is designated by c. The columns marked T and D will be explained later. The first five tables are for the combination crown-glass, air, crown-glass.

TABLE I.

Crown-glass, air, crown-glass. Red light, $\lambda = .618\,\mu$ *to* $.658\,\mu$. $\mu_1 = \mu = 1.5137$. $c = 41° 21'$.

i	ϕ	d		d/λ		T		D	
		//	$\perp$	//	$\perp$	//	T	Obs.	Cal.
5°45′	41°15′	0.78	1.55	1.20	2.43				
5 30	41 25	.98	2.15	1.54	3.33	2.68	4.43	1.79	2.54
5 15	41 35	.97	1.77	1.52	2.78	2.26	3.49	1.26	1.33
5	41 44	.90	1.55	1.41	2.43	2.03	2.97	1.02	1.00
3	43 4	.85	1.06	1.33	1.67	1.39	1.77	.34	.38
0	45 3	.71	.82	1.11	1.29	1.09	1.26	.18	.18
5	48 21	.55	.60	.86	.94	.87	.92	.08	.05
10	51 38	.49	.46	.77	.72	.76	.74	−.05	−.05
20	58 6	.39	.35	.61	.55	.60	.54	−.06	−.06
30	64 21	.30	.26	.47	.40	.50	.42	−.07	−.08

TABLE II.

Yellow light, $\lambda = .571\,\mu$ *to* $.608\,\mu$. $\mu = 1.5151$. $c = 41°18'$.

i	ϕ	d		d/λ		T		D	
		//	$\perp$	//	$\perp$	//	$\perp$	Obs.	Cal.
5°45′	41°15′	1.	1.71	1.70	2.90				
5 30	41 25	1.05	2.11	1.78	3.58	2.15	3.77	1.80	1.93
5 15	41 35	1.03	1.74	1.75	2.95	1.82	3.01	1.20	1.22
5	41 44	.99	1.54	1.68	2.61	1.78	2.63	.93	.93
3	43 4	.80	.98	1.35	1.66	1.26	1.63	.31	.37
0	45 3	.61	.71	1.03	1.20	.99	1.18	.17	.18
5	48 20	.48	.52	.82	.88	.81	.86	.06	.05
10	51 37	.41	.40	.69	.68	.70	.69	−.01	−.01
20	58 5	.34	.31	.58	.53	.56	.50	−.05	−.06
30	64 19	.27	.22	.46	.37	.47	.41	−.09	−.08

TABLE III.

Green light, $\lambda = .520\,\mu$ *to* $.550\,\mu$. $\mu = 1.5175$. $c = 41°12'$.

i	ϕ	d		d/λ		T		D	
		$\parallel$	$\perp$	$\parallel$	$\perp$	$\parallel$	$\perp$	Obs.	Cal.
5°45′	41°16′	0.92	1.73	1.72	3.24	2.30	4.68	1.52	3.95
5 30	41 25	.90	1.55	1.68	2.90	1.97	3.25	1.22	1.36
5 15	41 35	.89	1.46	1.66	2.73	1.78	2.76	1.07	1.01
5	41 44	.86	1.19	1.60	2.22	1.66	2.43	.62	.66
3	43 4	.66	.85	1.23	1.59	1.22	1.58	.36	.38
0	45 3	.54	.62	1.	1.16	.98	1.16	.16	.17
5	48 20	.45	.47	.84	.88	.79	.85	.04	.05
10	51 37	.37	.37	.68	.68	.68	.68	.00	−.00
20	58 4	.30	.27	.56	.51	.55	.49	−.05	−.06
30	64 17	.24	.20	.45	.37	.46	.38	−.08	−.08

TABLE IV.

Violet, $= .441\,\mu$ *to* $.466\,\mu$. $\mu = 1.5233$. $c = 41°2'$.

i	ϕ	d		d/λ		T		D	
		$\parallel$	$\perp$	$\parallel$	$\perp$	$\parallel$	$\perp$	Obs.	Cal.
5°45′	41°16′	0.61	1.23	1.35	2.71	1.22	2.24	1.36	1.34
5 30	41 26	.58	1.00	1.28	2.21	1.16	1.99	.93	.99
5 15	41 36	.54	.92	1.19	2.03	1.11	1.82	.84	.81
5	41 45	.49	.78	1.08	1.72	1.08	1.71	.64	.61
3	43 5	.38	.56	.84	1.23	.88	1.20	.39	.33
0	45 3	.30	.38	.66	.84	.74	.91	.18	.16

With the yellow light, of the same wave-length as in Table II., on the slit, a yellow glass, which was found by the usual photometric method to absorb approximately one-half the light of this wave-length, was placed in front of the collimator slit and the following results obtained.

TABLE V.

ϕ	d		d/λ		D		ϕ	d		d/λ		D	
	$\parallel$	$\perp$	$\parallel$	$\perp$	Obs.	Cal.		$\parallel$	$\perp$	$\parallel$	$\perp$	Obs.	Cal.
41°15′	0.83	1.33	1.40	2.26			48°21′	0.54	0.64	0.91	1.08	0.17	0.18
41 25	.84	1.67	1.42	2.84	1.42	1.93	57 37	.46	.49	.78	.83	.05	.05
41 35	.85	1.51	1.44	2.57	1.13	1.22	58 5	.38+	.38−	.65	.64	−.01	−.01
43 4	.81	1.34	1.37	2.28	.91	.93	64 19	.28	.24	.48	.41	−.07	−.06
45 3	.65	.90	1.10	1.52	.42	.37							

By comparing Table V. with Table II. where approximately twice the intensity was used, it appears that doubling the intensity increases the penetration in the ratio 1 : 1.18, the ratio being the same for either plane of polarization. This ratio is not constant however but varies with the magnitude of the original intensity.

No simple relation appears which would give the way in which the penetration varies with the intensity or with the wave-length of the incident light. Quincke states that the penetration varies almost, but not exactly, as the wave-length. This can be accepted only as a very rough approximation.

In working with white light it was noticed that the penetration for glycerine was approximately the same as for turpentine. Since the indices of refraction of these substances are approximately equal it would indicate, other conditions being the same, that the penetration is a function of the index of refraction alone. This point was tested by preparing a solution of common salt in water of such density as to give the same index of refraction as wood alcohol. Yellow light of the same wave-length as before was used.

TABLE VI.

Crown-glass, Wood-alcohol, Crown-glass. $\mu_2 = 1.3388$, $\mu = 1.1317$. $c = 62°5'$.						Crown-glass, Salt Solution, Crown-glass. $\mu_2 = 1.3388$. $\mu = 1.1317$. $c = 62°5'$.					
i	ϕ	d		d/λ		d		d/λ		D	
		∥	⊥	∥	⊥	∥	⊥	∥	⊥	Obs.	Cal.
26°30′	62°13′	1.60	1.99	3.64	4.52	1.64	2.03	3.71	4.58	0.87	0.87
26 45	62 21	1.63	1.92	3.69	4.35	1.62	1.93	3.67	4.37	.70	.59
27	62 30	1.57	1.78	3.55	4.04	1.56	1.80	3.54	4.07	.53	.47
28	63 7	1.28	1.40	2.90	3.18	1.27	1.38	2.88	3.13	.25	.27
30	64 20	1.04	1.10	2.36	2.49	1.02	1.08	2.31	2.45	.14	.15
40	70 10	.63	.64	1.42	1.45	.60	.61	1.36	1.38	.02	.02
50	75 27	.48	.46	1.08	1.04	.43	.40	.97	.91	−.06	−.01

The differences in the above table between the values for alcohol and the salt solution are easily within the range of experimental error. Since substances of such different chemical nature give the same values, the conclusion seems justifiable that the penetration is a function of the index of refraction alone, other conditions being the same.

TABLE VII.

Flint-glass, Air, Flint-glass. Red light, $\lambda = .618\ \mu$ *to* $.658\ \mu$. $\mu_1 = \mu = 1.6450$. $c = 37°26'$.

i	ϕ	d		d/λ		T		D	
		∥	⊥	∥	⊥	∥	⊥	Obs.	Cal.
12°30′	37°23′	0.75	1.51	1.18	2.36				
12 15	37 32	.82	2.01	1.30	3.16	1.66	3.49	1.86	1.92
12	37 41	.78	1.66	1.22	2.60	1.51	2.78	1.38	1.45
10	38 53	.72	1.02	1.13	1.60	1.10	1.57	.47	.48
5	41 54	.52	.65	.81	1.03	.80	.95	.22	.23
0	44 56	.44	.47	.69	.74	.68	.72	.05	.04
10	51 0	.34	.32	.53	.50	.53	.49	−.03	−.04
20	56 56	.29	.23	.45	.36	.45	.38	−.09	−.07
30	62 38	.25	.20	.39	.31	.38	.29	−.08	−.08
40	67 56	.20	.15	.32	.23	.31	.22	−.09	−.09

An interesting case is where the first and third media are different. Red light of the same wave-length as before was used.

TABLE VIII.

Crown-glass, Air, Flint-glass. $\mu = 1.5137$. $c = 41°21'$.					
i	ϕ	d		d/λ	
		∥	⊥	∥	⊥
5°45′	41°15′	0.84	1.61	1.31	2.52
5 30	41 25	.86	2.07	1.35	3.25
5 15	41 35	.84	1.64	1.32	2.57
5	41 44	.82	1.38	1.28	2.16
3	43 4	.67	.94	1.05	1.47
0	45 3	.61	.67	.96	1.05
10	51 38	.44	.40	.69	.63
20	58 6	.33	.28	.52	.44

TABLE IX.

Flint-glass, Air, Crown-glass. $\mu = 1.6450$. $c = 37°26'$.					
i	ϕ	d		d/λ	
		∥	⊥	∥	⊥
12°30′	37°23′	0.86	1.66	1.35	2.60
12 15	37 32	.89	2.09	1.39	3.28
12	37 41	.88	1.71	1.37	2.68
10	38 53	.70	1.10	1.10	1.73
5	41 54	.58	.65	.91	1.02
0	44 56	.51	.49	.80	.77
10	51 0	.38	.33	.60	.52

Plotting these results, Fig. 4, it appears that the curves for crown glass as first medium are approximately the same as when flint glass is the first medium excepting that they are displaced nearly four degrees to the right, that is, displaced by a distance equal to the difference between the critical angles. It will be seen from the tables that for values near the critical angles, the penetration when flint glass is first medium is slightly greater than when the crown glass is the first medium. This difference, however, is within experimental error. Furthermore since the angles of incidence do not

correspond, and also since different pairs of prisms are used in the two cases no conclusion can be drawn. To test this point more fully the flint glass prism with a plane hypotenuse was used with the crown-glass prism having the hypotenuse surface spherical. In the

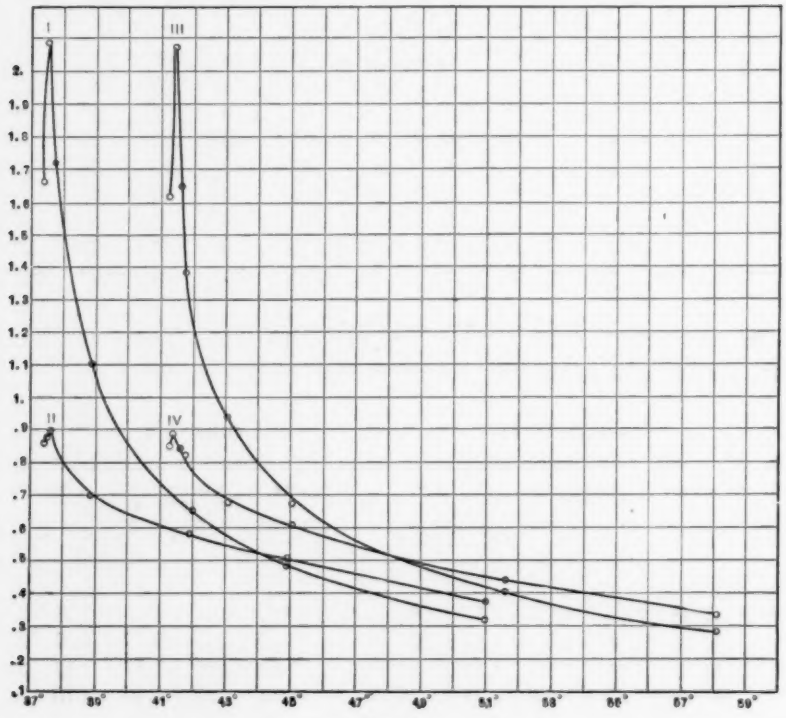


Fig. 4.

I., Flint-glass, air, crown-glass, light polarized perpendicular to plane of incidence; II., Flint-glass, air, crown-glass, light polarized in the plane of incidence; III., Crown-glass, air, flint-glass, light polarized perpendicular to plane of incidence; IV., Crown-glass, air, flint-glass, light polarized in the plane of incidence.

crown-glass prism the angle made by the side on which the light was incident with a plane tangent at the center of the convex surface was 45°. The results are given in Table X. Yellow light was used. The angles of incidence on the film, φ for the direction crown-glass, air, flint-glass and θ for the reverse direction, are made to correspond from the relation $\sin \varphi = \frac{\mu_C}{\mu_F} \sin \theta$.

The same calibration curve was used for both sets of measure-

TABLE X.

Crown-glass, Air, Flint-glass. $\mu = 1.5164$.						Flint-glass, Air, Crown-glass. $\mu = 1.6488$.					
i_1	ϕ	d		d/λ		i_2	θ	d		d/λ	
		$\parallel$	$\perp$	$\parallel$	$\perp$			$\parallel$	$\perp$	$\parallel$	$\perp$
5°	41°42′	0.96	1.40	1.62	2.38	11°58′30″	37°42′45″	0.98	1.45	1.66	2.46
3	43 1 20″	.87	.94	1.48	1.59	10 1	38 53 10	.90	.97	1.53	1.64
0	45	.75	.76	1.27	1.29	7 14 10	40 33 30	.77	.80	1.30	1.36
5	48 18	.60	.57	1.02	.97	2 36 20	43 21 30	.63	.58	1.07	.98

ments. The measurements are smaller throughout for the case where crown glass is the first medium. A water film also gave results slightly greater for the case where flint glass was the first medium. Although the differences are small and probably not in excess of experimental error, the consistency of the differences in both the case of a water and an air film make it seem highly probable that the penetration is greater for the direction flint glass film crown glass than for the reverse direction. This conclusion agrees with Quincke[1] although the difference is much less than his results show. The method he employed, which assumed that the prisms were always just in contact, is open to too great error to have any value on this point.

By comparing Tables VIII. with I., and IX. with VII., it will be noticed that the values obtained for crown-glass, air, flint-glass or flint-glass, air, crown-glass lie between those obtained for crown-glass, air, crown-glass and flint-glass, air, flint-glass. The same relation is brought out in the following tables, where yellow light of the same wave-length as before was used.

TABLE XI.

Flint-glass, Benzole, Flint-glass. $\mu_1 = 1.6488$. $\mu_2 = 1.4998$. $\mu = 1.0994$. $c = 65°27'$.					
i	ϕ	d		d/λ	
		$\parallel$	$\perp$	$\parallel$	$\perp$
35°30′	65°34′	1.61	2.21	4.10	5.63
36	65 49	1.48	1.82	3.77	4.64
38	66 52	1.03	1.24	2.62	3.16
40	67 53	.89	.99	2.26	2.52
45	70 20	.62	.62	1.78	1.58

TABLE XII.

Flint-glass, Benzole, Crown-glass. $\mu = 1.0994$. $c = 65°27'$.				
ϕ	d		d/λ	
	$\parallel$	$\perp$	$\parallel$	$\perp$
65°34′	2.13	2.50	5.42	6.25
65 49	1.82	2.03	4.64	5.16
66 52	1.10	1.28	2.80	3.26

[1] Loc. cit., p. 16.

TABLE XIII.

Crown-glass, Benzole, Crown-glass.

$\mu_1 = 1.5151$. $\mu_2 = 1.4998$. $\mu = 1.0102$. $c = 81°51'$.

i	ϕ	d		d/λ		T		D	
		//	⊥	//	⊥	//	⊥	Obs.	Cal.
12°	82°6′	2.68	2.73	6.83	6.95	6.70	6.81	0.12	0.10
11 30′	82 25	2.29	2.40	5.83	6.11	5.56	5.63	.28	.06
11	82 45	2.16	2.22	5.50	5.65	4.87	4.92	.15	.04
10	83 24	1.65	1.72	4.20	4.38	4.06	4.09	.18	.02
9	84 5	1.40	1.44	3.56	3.67	3.55	3.56	.11	.01
7	85 22	1.12	1.11	2.85	2.83	2.87	2.86	−.02	−.01−
5	86 41	.90	.88	2.32	2.30	2.32	2.30	−.02	−.01+

The measurements are exceedingly hard to obtain when the relative index of refraction is as small as in the case of crown glass and benzole. The percentage of error is therefore high, individual measurements varying from 10 to 15 per cent. from the average. The arrangement of the prisms was the same as shown in Fig. 2, the "right angle" of the prism on which the light was incident being 89° 59′ 5″. The intensity would be slightly greater for Table XIII. than for Tables XI. and XII., but allowing for this, the penetration is still greater for crown glass-benzole-crown glass than for flint glass-benzole-crown glass and this in turn is greater than for flint glass-benzole-flint glass. It follows from this that as the index of refraction of the third medium increases, the distance to which the light vibration penetrates the film decreases; and as the index of refraction of the third medium decreases this distance increases. It was pointed out before that as the relative index of refraction between the first and second mediums decrease the penetration increases. The same relation therefore exists between the second and third mediums as between the first and second.

It would be interesting to know to what extent the penetration depends upon the layer of carbon dioxide, water vapor, etc., condensed upon the surface of the glass. There seems to be no method of getting rid of this layer except by heating to redness for a considerable length of time, a method hardly practical in this case.

For an air film measurements taken after the prisms had been in a dry vacuum for two days agreed with measurements taken after

the prisms had been for nearly the same length of time in moist carbon dioxide under a pressure slightly greater than atmospheric pressure. It is doubtful whether any change in the layer of condensed gases was produced by this treatment.

Strongly Absorbing Medium.

The penetration was next examined in the film of a substance showing anomalous dispersion and hence having a strong absorp-

Table XIV.

Flint-glass, Fuchsine, Flint-glass.

Red $\lambda = .620\,\mu$ to $.658\,\mu$. $\mu_1 = 1.6450$. $\mu_2 = 1.389$. $\mu = 1.184$. $c = 57°36'$.

i	ϕ	d		d/λ	
		$\parallel$	$\perp$	$\parallel$	$\perp$
21°	57°31′	1.17	1.28	2.54	2.78
21 15′	57 40	1.33	1.66	2.89	3.61
21 30	57 49	1.18	1.61	2.56	3.50
22	58 6	1.10	1.42	2.39	3.09
23	58 41	.98	1.28	2.13	2.78
24	59 15	.91	1.06	1.98	2.30
25	59 50	.88	.99	1.91	2.15
30	62 38	.72	.77	1.56	1.67
35	65 21	.63	.60	1.37	1.31
40	67 56	.50	.48	1.09	1.04
45	70 24	.43	.41	.94	.89

Yellow $\lambda = .579\,\mu$ to $.612\,\mu$. $\mu_1 = 1.6488$. $\mu_2 = 1.412$. $\mu = 1.167$. $c = 58°55'$.

i	ϕ	d		d/λ	
		$\parallel$	$\perp$	$\parallel$	$\perp$
23°	58°39′	1.37	1.85	3.26	4.40
23 30′	58 56	1.39	2.18	3.31	5.18
24	59 13	1.28	1.66	3.04	3.95
25	59 47	1.17	1.25	2.78	2.97
30	62 36	.87	.86	2.06	2.04
35	65 18	.68	.65	1.61	1.54
40	67 53	.55	.51	1.30	1.21
45	70 20	.45	.43	1.07	1.02

Green $\lambda = .513\,\mu$ to $.548\,\mu$. $\mu_1 = 1.655$. $\mu_2 = 1.383$. $\mu = 1.197$. $c = 56°41'$.

i	ϕ	d		d/λ	
		$\parallel$	$\perp$	$\parallel$	$\perp$
20°	33° 1′	1.80	1.85	4.70	4.83
0	44 56	1.31	1.41	3.42	3.68
10	50 49	.97	1.05	2.53	2.74
18	55 42	.65	.73	1.70	1.90
20	56 52	.51	.55	1.33	1.43
21	57 27	.53	.51	1.38	1.33
22	58 1	.49	.51	1.28	1.33
23	58 36	.47	.50	1.23	1.30
24	59 10	.46	.48	1.20	1.25
25	59 44	.43	.44	1.12	1.15
30	62 31	.39	.39	1.02	1.02
35	65 13	.35	.34	.91	.89
40	67 48	.33	.32	.86	.84
45	70 14	.28	.27	.73	.71

Blue $\lambda = .440\,\mu$ to $.458\,\mu$. $\mu_1 = 1.664$. $\mu_2 = 1.372$. $\mu = 1.212$. $c = 55°32'$.

i	ϕ	d		d/λ	
		$\parallel$	$\perp$	$\parallel$	$\perp$
21°	57°23′	0.40	0.67	1.22	2.23
21 30′	57 40	.37	.54	1.13	1.67
22	57 57	.36	.46	1.10	1.41
23	58 31	.35	.41	1.07	1.25
24	59 5	.33	.39	1.01	1.19
25	59 39	.34	.38	1.04	1.16
30	62 25	.28	.30	.86	.92
35	65 6	.25	.24	.76	.73
40	67 37	.23	.22	.70	.67

tion band. A 7 per cent. solution of fuchsine in ethyl alcohol was used. The fuchsine was manufactured by the Heller and Merz Co.

The indices of refraction were obtained with a prism of the liquid, the prism angle varying from a few degrees for the red to two minutes for the green. For red and yellow light the slit width was 0.75 mm., and for the green and blue light the slit width was 1.2 mm.

These values do not differ in their qualitative relations from those where the film is not strongly absorbent. This is true even for the green where the measurements were made for the region including the absorption band. The values are numerically smaller than those obtained with an alcohol film, as might be expected. This is especially so in the case for the green light. It will also be noticed that for the green light the diameter of the transmitted spot of light does not decrease as the angle of incidence decreases from the critical angle. This is due to the fact that no interference bands appear in the transmitted light, on account of the strong absorption, so that what is measured in this case is not the diameter of the inner spot of light but the diameter corresponding to the total effect of all the transmitted light. In the reflected system one interference band was distinctly visible, which disappeared as the critical angle was reached.

PENETRATION WITH TWO MEDIA.

It has been shown that as the optical density of the third medium decreases the penetration increases. If it be allowable to extend this reasoning, it would appear that if the third medium became optically homogeneous with the second, the penetration would be greater than for any case in which the third medium was denser than the second. This is equivalent to saying that the penetration is greater where the second medium is thick, and hence only two media are concerned, than it is when the second medium is a thin film enclosed between denser substances.

To show that penetration does occur where only two media are involved the following method was employed. Sensitized gelatine was prepared from the formula given by Valenta for the Lippman

process of color photography. The silver bromide grains were exceedingly fine and the emulsion fairly transparent. One side of a 60° flint-glass prism was covered with a heavy coat of emulsion. When the coat was nearly dry another was added, thus giving a layer of gelatine of great thickness as compared with the penetration phenomena. A beam of sunlight, nearly one centimeter in diameter was introduced into a dark room and allowed to fall upon the prism so that the light passing through the prism would be incident on the emulsion at an angle greater than the critical angle. The prism was in a small black box; the totally reflected light passing out of a small opening, made for the purpose, was absorbed by the black walls of the room. An exposure of five minutes gave, after development, a sharply defined black spot of the size of the incident beam. The film was transferred from the prism to a paraffine block. The darkened spot was on the side of the film next the prism and other parts of the film gave no evidence of exposure. The depth of this darkening was about 0.005 mm. This was obtained by examining a cross-section of the film under a high-power microscope. The fact that the spot was sharply defined shows that the effect was not due to light diffused from the first surface of the prism. That the effect is not due to the light diffused from the second surface is shown by the very small depth to which the darkening extends. As a further proof on the last point the prism was cleaned and placed in the same position as when the exposure was made. A glass plate coated with emulsion was placed about one fourth of a millimeter back of the prism surface from which the light was totally reflected. An exposure of the same length as before failed to show the slightest effect on the sensitive plate. The darkening obtained cannot be due to the light diffused from the surfaces of the prism.

The emulsion may be regarded as gelatine holding in suspension grains of silver bromide. The average diameter of these grains, as obtained with a high-power microscope with a micrometer eyepiece, was less than a wave-length of light. Since only a portion of the surface of the grain could be in contact with the glass, it would not seem probable that the effect is due to those grains which happen to be in actual contact with the glass. That the effect is not due to

this cause is clearly shown by the following procedure. One surface of the prism was wet with a very dilute solution of collodion in ether, thus a very thin film of collodion was obtained. The prism was cooled and the emulsion applied to the collodion surface. An exposure, as above, was made, total reflection taking place at the boundary between the glass and the collodion. A dark spot was again obtained on the sensitized gelatine. The film was removed from the prism, under water, the collodion coming off with the gelatine. The collodion film was partially separated from the gelatine, while under water, that portion of the film over the darkened spot was clear, hence no grains of silver bromide had penetrated it. Where the collodion film became several wave-lengths thick no effect was produced in the sensitized gelatine.

The photographic effect, then, must be due to the energy which has passed into the second medium.

Voigt [1] has given a method for showing, without the help of a third body, that in total reflection the vibrations enter the second medium. From a rectangular prism *efg*, Fig. 5, a wedge *afb* is cut. The surfaces *ba* and *ae* are carefully ground, the edge *a* being cylindrical. If light be allowed to fall nearly normally upon *gb*, so as to be totally reflected from the surfaces *ba* and *ae*, and then if one looks at *a* from some point in *oh*, the edge *a* appears as a streak of light. The intensity of this light decreases rapidly as the angle of incidence is increased, and other characteristics peculiar to penetration phenomena are noted. In support of the experiment Voigt advances theoretical considerations which have been called in question by Ketteler.[2]

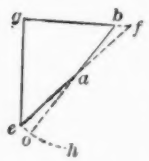


Fig. 5.

ELLIPTICAL POLARIZATION.

The two prisms were clamped together as they were when the penetration distances were measured. The transmitted and reflected light was then examined with a Babinet compensator and the following conclusions drawn. The incident light was polarized in the azimuth 45°.

[1] Wied. Ann., 67, 1899, p. 185.
[2] Wied. Ann., 67, 1899, p. 879. For Voigt's reply see Wied. Ann., 68, p. 135.

1. If the incident parallel light be plane-polarized in any azimuth, both the transmitted and "totally reflected" light will be elliptically polarized.

2. The phase difference, in both the transmitted and reflected light, is independent of the intensity.

3. The phase difference decreases as the index of refraction of the film increases.

4. The phase difference is the same whether the order be flint glass-air-crown glass or crown glass-air-flint glass.

5. The phase difference in the transmitted system increases with an increase in the thickness of the layer penetrated.

6. As the angle of incidence increases, the phase difference increases from zero at the critical angle to a maximum, and then decreases to zero for an angle of incidence of 90°. The maximum value occurs at the angle of incidence for which the distance of penetration is independent of the plane of polarization. That is, at the angle of incidence corresponding to the intersection of the two curves representing the penetration as given in Fig. 3.

6. The direction of the axes of polarization in the transmitted light are parallel and perpendicular to the plane of polarization of the incident light.

The above conclusions are to be considered simply as very close approximations.

THEORETICAL.

We think of light energy as being propagated by means of ether waves, whatever may be the substance through which propagation takes place. That there is a discontinuity in the condition of the ether at the geometrical boundary between two bodies, or between a body and free space, is hardly conceivable. Therefore whatever may be the difference in the behavior of the ether in two different media, the change in this behavior as one passes from one medium to the other must be a gradual, that is, a continuous change. We should therefore expect, in the case of total reflection, some such phenomena as have been experimentally observed.

For the case where a thin film is enclosed between two denser media Voigt[1] has given a very complete theory. Voigt's theory

[1] Nachrichten v. d. Koniglichen Gesellschaft d. Wissenschaft Göttingen, 1884, p. 49.

has for its starting point the equations of reflection and refraction as deduced from the Neumann and MacCullagh theory, combined with the principle "that in the boundary between two different media a certain work between the ponderable and ether particles must be done by the active vibrating force."[1] Voigt shows the equations he derives to be in qualitative accord with the experimental results of Quincke, with one exception. Quincke states that the penetration is greater when the light passes through the prisms in the direction flint glass-air-crown glass than when the order is reversed. According to Voigt's theory the penetration should be the same in the two cases.

Charles Fabry,[2] starting with the equations for interference in thin films, has also developed equations for the case where the first and third media are the same.

Drude, in Winkelmann's Handbuck der Physik,[3] gives a short discussion of total reflection in thin films, the first and third media being the same.

The electromagnetic theory of light affords a method of deriving equations which agree, both qualitatively and quantitatively, with experimental results. In the following development the symbols used are in accord with those used in Drude's Lehrbuch der Optik. Let the three media be designated 1, 2, 3, with dielectric constants ε_1, ε_2, ε_3, respectively. The index of refraction from 1 to 2 is then given by the equation $n_1 = \frac{\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1}}$, and from 2 to 3 by the equation $n_2 = \frac{\sqrt{\varepsilon_3}}{\sqrt{\varepsilon_2}}$. We have then $n_1 < 1 < n_2$. Let the plane of incidence be the plane of the paper, or the XZ plane; the XY plane being the boundary between media 1 and 2, and the plane $z = d$ the boundary between media 2 and 3. X, Y, Z are the components of electric force parallel respectively to the axes x, y, z; α, β, γ, are the corresponding components of the magnetic force. Let the incident wave be a plane parallel wave whose angle of incidence on the boundary 1–2 is φ, the angle of refraction in 2 and 3 being ψ and θ respectively. E, R and D are, respectively, the electric forces in the incident, re-

[1] G. Kirchhoff, Berlin Acad., 1876, p. 75.
[2] Compte Rendus, t. 120, 1895, p. 314.
[3] B. II., 1, p. 780.

flected, and transmitted wave. The electric force will be considered as broken up into two components, one perpendicular and the other parallel to the plane of incidence, distinguishing between them by the subscripts s and p respectively. For the case where the electric force is perpendicular to the plane of incidence we have the real parts of the following expressions.

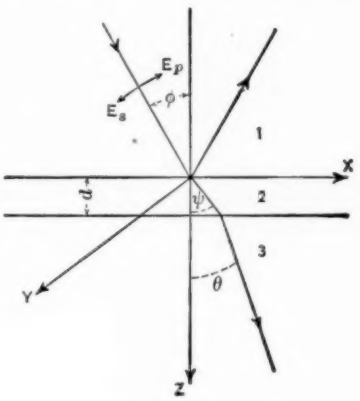


Fig. 6.

For the incident wave in the first medium,[1]

$$Y_s = E_s e^{i\frac{2\pi}{T}\left(t-\frac{x\sin\phi+z\cos\phi}{v_1}\right)}$$

$$(A)\quad a_s = -E_s\cos\varphi\,\sqrt{\varepsilon_1}\,e^{i\frac{2\pi}{T}\left(t-\frac{x\sin\phi+z\cos\phi}{v_1}\right)}$$

$$\gamma_s = E_s\sin\varphi\,\sqrt{\varepsilon_1}\,e^{i\frac{2\pi}{T}\left(t-\frac{x\sin\phi+z\cos\phi}{v_1}\right)}.$$

For the reflected wave in the first medium,

$$Y_r = R_s e^{i\frac{2\pi}{T}\left(t-\frac{x\sin\phi-z\cos\phi}{v_1}\right)}$$

$$(B)\quad a_r = R_s\cos\varphi\,\sqrt{\varepsilon_1}\,e^{i\frac{2\pi}{T}\left(t-\frac{x\sin\phi-z\cos\phi}{v_1}\right)}$$

$$\gamma_r = R_s\sin\varphi\,\sqrt{\varepsilon_1}\,e^{i\frac{2\pi}{T}\left(t-\frac{x\sin\phi-z\cos\phi}{v_1}\right)}.$$

For the wave moving in the second medium so that z is increasing.[2]

$$Y' = D_s' e^{i\frac{2\pi}{T}\left(t-\frac{x\sin\psi+z\cos\psi}{v_2}\right)}$$

$$(C)\quad a' = -D_s'\cos\psi\,\sqrt{\varepsilon_2}\,e^{i\frac{2\pi}{T}\left(t-\frac{x\sin\psi+z\cos\psi}{v_2}\right)}$$

$$\gamma' = D_s'\sin\psi\,\sqrt{\varepsilon_2}\,e^{i\frac{2\pi}{T}\left(t-\frac{x\sin\psi+z\cos\psi}{v_2}\right)}.$$

For the wave moving in the second medium so that z is decreasing,

[1] Drude, Lehrbuch der Optik, p. 258.

[2] Equations C and D are summations. They take account of the multiple reflections, the phase differences introduced being included in the amplitudes D_s' and D_s'' which are to be regarded as complex.

$$Y'' = D_s'' e^{i\frac{2\pi}{T}\left(t - \frac{x\sin\psi - z\cos\psi}{v_2}\right)}$$

$$(D) \qquad \alpha'' = D_s'' \cos\psi \sqrt{\varepsilon_2}\, e^{i\frac{2\pi}{T}\left(t - \frac{x\sin\psi - z\cos\psi}{v_2}\right)}$$

$$\gamma'' = D_s'' \sin\psi \sqrt{\varepsilon_2}\, e^{i\frac{2\pi}{T}\left(t - \frac{x\sin\psi - z\cos\psi}{v_2}\right)}.$$

For the wave moving in the third medium,

$$Y_3 = D_s e^{i\frac{2\pi}{T}\left(t - \frac{x\sin\theta + z\cos\theta}{v_3}\right)},$$

$$(E) \qquad \alpha_3 = -D_s \cos\theta \sqrt{\varepsilon_3}\, e^{i\frac{2\pi}{T}\left(t - \frac{x\sin\theta + z\cos\theta}{v_3}\right)},$$

$$\gamma_3 = D_s \sin\theta \sqrt{\varepsilon_3}\, e^{i\frac{2\pi}{T}\left(t - \frac{x\sin\theta + z\cos\theta}{v_3}\right)}.$$

The boundary conditions are :

(I) $Y_s + Y_r = Y' + Y''$, $\alpha_e + \alpha_r = \alpha' + \alpha''$, $\gamma_e + \gamma_r = \gamma' + \gamma''$ for $z = 0$,

(II) $Y' + Y'' = Y_3$, $\quad \alpha' + \alpha'' = \alpha_3$, $\quad \gamma' + \gamma'' = \gamma_3$ $\quad$ for $z = d$.

Substituting in these equations of boundary condition the value of Y_s, Y_r, etc., as obtained from equations A to E inclusive we have, from I,

(1) $$E_s + R_s = D_s' + D_s'',$$

(2) $$E_s - R_s = n_1(D_s' - D_s'')\frac{\cos\psi}{\cos\varphi},$$

and from (II.), letting

$$u = i\frac{2\pi d}{\lambda_2}\cos\psi \quad \text{and} \quad q = \frac{2\pi d}{\lambda_3}\cos\theta,$$

(3) $$D_s' e^{-u} + D_s'' e^{u} = D_s e^{-iq},$$

(4) $$D_s' e^{-u} - D_s'' e^{u} = D_s e^{-iq} \cdot n_2 \frac{\cos\theta}{\cos\psi}.$$

When φ becomes greater than the critical angle ψ becomes imaginary. Under these conditions we may write, since $\sin\psi = \sin\varphi/n_1$

(5) $$\cos\psi = -i\sqrt{\frac{\sin^2\varphi}{n_1^2} - 1}.^1$$

[1] Cos ψ is negative imaginary, since otherwise the amplitude of vibration would increase with an increase in the distance of penetration, as will be seen from later equations.

If we introduce the relation (5) in our equations the amplitude D_s becomes complex. The physical meaning of a complex amplitude is that a phase change has been introduced.[1] We may write the relation then $D_s = D_s e^{i\delta}$ where D is real and δ is the phase difference between the transmitted and incident light. u also takes a new value and is no longer imaginary.

$$u = i\frac{2\pi d}{\lambda_2}\left(-i\sqrt{\frac{\sin^2\varphi}{n_1^2}-1}\right) = \frac{2\pi}{n_1}\frac{d}{\lambda_2}\sqrt{\sin^2\varphi - n_1^2}.$$

Eliminating D_s', D_s'', and R_s from equations 1, 2, 3, and 4, and introducing the relation (5) we have

$$(6)\quad D_s e^{i\delta}e^{-i\eta} = E_s \cdot \frac{4i\cos\varphi\sqrt{\sin^2\varphi - n_1^2}}{(e^u + e^{-u})(\cos\varphi + n_1 n_2\cos\theta)\, i\sqrt{\sin^2\varphi - n_0^2} + (e^u - e^{-u})[\sin^2\varphi - n_1^2 - n_1 n_2\cos\varphi\cos\theta]^2}.$$

Multiplying this equation by its conjugate imaginary gives

$$(7)\quad D_s^2 = E_s^2 \cdot \frac{4\cos^2\varphi\,(\sin^2\varphi - n_1^2)}{(1-n_1^2)(\sin^2\varphi + n_1^2 n_2^2\cos^2\theta - n_1^2)\sinh^2 u + (\cos\varphi + n_1 n_2\cos\theta)(\sin^2\varphi - n_1^2)}.$$

A similar proceeding gives for the square of the amplitude in the third medium when the incident light is polarized so that the electric force is in the plane of incidence,

$$(8)\quad D_p^2 = E_p^2 \frac{4n_1^2\cos^2\varphi\,(\sin^2\varphi - n_1^2)}{(1-n_1^2)(\sin^2\varphi - n_1^2\cos^2\varphi)[n_2^2(\sin^2\varphi - n_1^2) + n_1^2\cos^2\theta]\sinh^2 u + n_1^2(n_1 n_2\cos\varphi + \cos\theta)^2(\sin^2\varphi - n_1^2)}.$$

The numerator of either (7) or (8) is always small. If d, which enters the expression for u, exceeds a very small value, the denominator becomes practically infinite, and hence there is no vibration in the third medium.

If the third medium is the same as the first $\theta = \varphi$ and $n_2 = 1/n_1$. Equations (7) and (8) then become

$$(9)\quad \frac{D_s^2}{E_s^2} = \frac{4\cos^2\varphi\,(\sin^2\varphi - n_1^2)}{(1-n_1^2)^2\sinh^2 u + 4\cos^2\varphi(\sin^2\varphi - n_1^2)}$$

$$(10)\quad \frac{D_p^2}{E_p^2} = \frac{4n_1^4\cos^2\varphi\,(\sin^2\varphi - n_1^2)}{(1-n_1^2)^2(\sin^2\varphi - n_1^2\cos^2\varphi)^2\sinh^2 u + 4n_1^4\cos^2\varphi\,(\sin^2\varphi - n_1^2)}.$$

[1] Lehrbuch der Optik, p. 268.

It remains to be shown that these equations are in complete accord with experiment. The conditions of the experiment are such that the intensity incident on the surface of the first prism is a constant, independent of the plane of polarization and the angle of incidence. Also the intensity observed in the transmitted light is not the intensity in the third medium, but the intensity after the light has left the third medium. Denoting the amplitudes in air by brackets we have

$$E_s^2 = (E_s)^2 \cdot s_1, \quad E_p^2 = (E_p)^2 \cdot p_1, \quad (D_s)^2 = D_s^2 \cdot s_3, \quad (D_p)^2 = D_p^2 \cdot p_3$$

where

$$s_1 = \frac{\sin 2i \sin 2r}{\sin^2 (i + r)}, \quad p_1 = \frac{\sin 2i \sin 2r}{\sin^2 (i + r) \cos^2 (i - r)}$$

and similarly for s_3 and p_3 at the surface between the third medium and air.

1. There is nothing periodic about equations (7) and (8) and they therefore show that there can be no interference rings for angles of incidence greater than the critical angle.

2. If we divide both numerator and denominator of the second member of equations (7) and (8) by $\sin^2 \varphi - n_1^2$ we shall have as a factor of one term in the denominator,

$$\frac{\sinh^2 \left\{ \frac{2\pi}{n_1} \frac{d}{\lambda} \sqrt{\sin^2 \varphi - n_1^2} \right\}}{\sin^2 \varphi - n_1^2}.$$

The value of this expression is a minimum when $\sin \varphi = n_1$, that is, at the critical angle and increases as φ increases. It follows therefore that the amplitude, and hence the depth of penetration in the second medium, is a maximum at the critical angle, and decreases as the angle of incidence increases.

3. θ decreases when n_2 increases. It follows then from (7) and (8) that as the index of refraction of the third medium increases the amplitude, and hence the penetration in the second medium, decreases and vice versa.

4. The wave-length, λ, occurs directly in the equations and also indirectly in the indices of refraction. The penetration decreases with decreasing wave-length.

5. We have the relation $\sin \varphi = n_1 n_2 \sin \theta$. Inserting this condition, and also introducing the corrective factors s_1 and s_3, equation (7) may be reduced to

$$(11) \quad \frac{(D_s)^2}{(E_s)^2} = s_1 s_3 \frac{4 \cos^2 \varphi \left(\frac{\sin^2 \varphi}{n_1^2} - 1 \right)}{(1 - n_1^2)(n_2^2 - 1) \sinh^2 u + (\cos \varphi - n_1 n_2 \cos \theta)^2 \left(\frac{\sin^2 \varphi}{n_1^2} - 1 \right)}.$$

If now the beam of light be reversed in direction and become incident at the angle θ on what has been designated the third medium, the factors s_1 and s_3 and also φ and θ will be interchanged. n_1 must now be replaced by $1/n_2$ and n_2 by $1/n_1$. Inserting these changes in (11) we have for the equation expressing the condition where light passes from the third to the first medium,

$$(12) \quad \frac{(D_s)^2}{(E_s)_2} = s_3 s_1 \frac{4 n_1^2 n_2^2 \cos^2 \theta (n_2 \sin \theta - 1)}{(1 - n_1^2)(n_2^2 - 1) \sinh^2 u' + (n_1 n_2 \cos \theta + \cos \varphi)^2 (n_2^2 \sin^2 \theta - 1)}.$$

The denominators of (11) and (12) are equal since

$$n_2^2 \sin^2 \theta = \frac{\sin^2 \varphi}{n_1^2}.$$

which relation makes $u = u'$. The numerators become the same if $\varphi = \theta$, that is, if the third and first media have the same optical density.

If the third medium is optically denser than the first we have $\cos \varphi < n_1 n_2 \cos \theta$. It follows therefore that where the first and third media are different the penetration is greater when the direction is such that the light passes from the denser to the rarer medium. Equation (8) leads to the same result.

For the case of an air film between crown glass and flint glass a value for $(D_s)^2/(E_s)^2$ was assumed and the values for d computed. The results as calculated from equation (12) were only a few hundredths of a wave-length greater than those obtained from equation (11).

This conclusion is directly opposed to Voigt's theory. Voigt's theory leads to the conclusion that the phenomenon is completely reversible. The path through the prisms is clearly reversible, but it is by no means evident that the intensities are independent of the direction. Furthermore, experiment seems to confirm the conclusion that the penetration and hence the intensities are not independent of the direction. Voigt's equations, in the form in which they are given, are complicated when expressed in terms of the indices of refraction and the angles involved. They may however be reduced to the form of (7) and (8) and become identical with (7) and (8) if the second member be multiplied by $n_1 n_2 \cos\theta/\cos\varphi$. This factor reduces to unity when the first and third media are the same and hence for this special case Voigt's equations become identical with (9) and (10).

6. Assuming that the sensitiveness of the eye does not change throughout a series of readings, the conditions of experiment are such that the amplitudes at the edge of the transmitted spot of light are constant. They are the amplitudes which just fail to produce an effect on the retina. Since the intensity of the source is assumed constant, we have, for the edge of the transmitted spot of light for any set of readings

$$\frac{(D_s)^2}{(E_s)^2} = \frac{(D_p)^2}{(E_p)^2} = C,$$

where C is some constant.

Rewriting equations (9) and (10) inserting the factors s_1 and p_1,

$$(9') \quad C = s_1^2 \frac{4\cos^2\varphi(\sin^2\varphi - n_1^2)}{(1-n_1^2)^2 \sinh^2 u_s + 4\cos^2\varphi(\sin^2\varphi - n_1^2)},$$

$$(10') \quad C = p_1^2 \frac{4n_1^4\cos^2\varphi(\sin^2\varphi - n_1^2)}{(1-n_1^2)^2(\sin^2\varphi - n_1^2\cos^2\varphi)^2 \sinh^2 u_p + 4n_1^4\cos^2\varphi(\sin^2\varphi - n_1^2)}.$$

It is to be remembered that equations (9') and (10') hold only for the edge of the transmitted spot of light. If we multiply both numerator and denominator of (9') by n_1^4 and examine the denominators of (9') and (10') we have

$$n_1^4 > [\sin^2\varphi - n_1^2\cos^2\varphi]^2 \quad \text{for} \quad \sin\varphi = n_1,$$

and

$$n_1^4 < [\sin^2\varphi - n_1^2\cos^2\varphi]^2 \quad \text{for} \quad \sin\varphi = 1.$$

That is, the penetration for increasing angle of incidence is at first greater for light polarized perpendicular to the plane of incidence, and later greater for light polarized parallel to the plane of incidence. To find the condition that $d_s = d_p$ we have, since the second term in the denominator is small in comparison with the first and hence can be neglected,

$$p_1^2 n_1^4 = s_1^2(\sin^2\varphi - n_1^2\cos^2\varphi)^2$$

which becomes

$$\sin^2\varphi = \frac{n_1^2(s_1 + p_1)}{s_1(1 + n_1^2)}, \tag{13}$$

or considering simply the three media

$$\sin^2\varphi = \frac{2n_1^2}{1 + n_1^2}.$$

This formula is in close agreement with experiment. It gives the values of φ for the intersection of the curves in Fig. 3.

7. The theory of elliptical polarization has been fully developed.[1] It can be deduced from the equations given and from a similar pair expressing the conditions in the reflected system. It is interesting to note that if the incident light be polarized in the azimuth of 45°, the phase difference between the components parallel and perpendicular to the plane of incidence, in both the reflected and transmitted systems, attains a maximum value at the value of φ given by equation (13).

8. The second term in the denominator of either (9′) or (10′) is small in comparison with the first term, and may be neglected. Dividing equation (9′) by (10′) we have, after extracting the square root,

$$p_1 n_1^2 \sinh u_s = s_1(\sin^2\varphi - n_1^2\cos^2\varphi)\sinh u_p.$$

Since in general the values of n are not small, we may neglect e^{-u} in comparison with e^{u} and obtain, on substituting the values of u

$$\left(u = \frac{2\pi}{n_1}\cdot\frac{d}{\lambda}\sqrt{\sin^2\varphi - n_1^2}\right),$$

$$\frac{d_p}{\lambda_2} - \frac{d_s}{\lambda_2} = \frac{\log\text{ nat}\left\{\sin^2\varphi\left(\frac{1 + n_1^2}{n_1^2}\right) - 1\right\}\frac{p}{s}}{\frac{2\pi}{n_1}\sqrt{\sin^2\varphi - n_1^2}}. \tag{14}$$

[1] Wüllner, Experimentalphysik, Vol. 4, p. 732.

This gives an approximate formula for testing experimental results. The formula is identical with that given by Voigt. In the experimental results in the columns marked D are given the values of $\frac{d_p}{\lambda_2} - \frac{d_s}{\lambda_2}$ as observed experimentally, and as calculated from equation (14). The agreement between the observed and calculated values is very close, excepting near the critical angle, where the approximations made are not justifiable. Near the critical angle $\sqrt{\sin^2\varphi - n_1^2}$ is small and hence e^{-u} cannot be neglected in comparison with e^{u}. The introduction of this approximation gives values higher than would otherwise be obtained. This fully explains the apparent discrepancy near the critical angles.

9. If the intensity of the incident light be increased to $\varkappa$ times the original intensity, then we have, neglecting the second term of the denominator,

$$C = \varkappa s_1^2 \frac{4\cos^2\varphi(\sin^2\varphi - n_1^2)}{(1 - n_1^2)^2 \sinh^2 u_{s\cdot\varkappa}}. \tag{15}$$

Dividing equation (9′) by (15) and neglecting e^{-u}, we have

$$\frac{d_{s\varkappa}}{\lambda} - \frac{d_s}{\lambda} = \frac{\log \text{nat}\sqrt{\varkappa}}{\frac{2\pi}{n_1}\sqrt{\sin^2\varphi - n_1^2}}. \tag{16}$$

The values for the differences in penetration obtained from this formula agree rather poorly from those obtained experimentally in Tables II. and V. This is probably due to the fact that the ratio of the intensities was only approximately obtained.

10. If we assume some value for d the ratio $(D_s)^2/(E_s)^2$ can be determined. The results in the preceding tables were plotted and the value of d at the intersections of the curves, where $d_s = d_p$, was assumed to be correct. From this value of d the ratio of the intensities was computed. Knowing this ratio the value of d for any angle of incidence could be calculated. Equations (9′) and (10′) were used, hence no approximations introduced. The columns marked T in the preceding tables give the calculated results. The results are also plotted in Figs. 7 and 8, the dotted lines being the computed curves, the observed values being denoted by circles (o) and crosses (×), as in Fig. 3. In all cases, with the exceptions

perhaps of Tables I. and III., the agreement between the calculated and observed results is, considering the severity of the test, remarkably close. The agreement is equally good whether the film be air, salt solution, or benzole. Near the critical angle the observed values are all less than the calculated values with

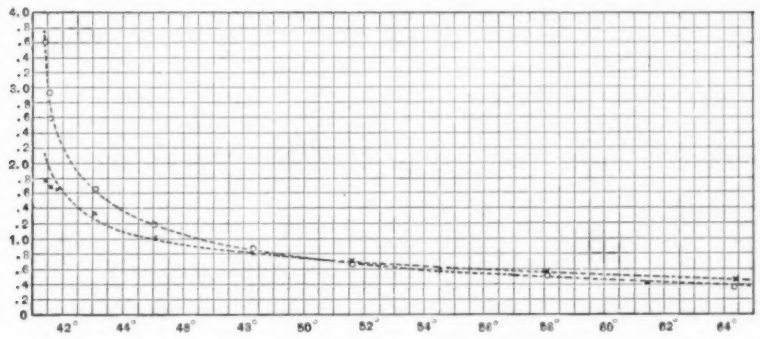


Fig. 7.

Crown glass air crown glass, Table II.

the exception of those for benzole. This is to be expected. Measurements near the critical angle are difficult to obtain, the transmitted spot of light spreads out and the edges become very diffuse. In making settings on this diffuse edge at a point where

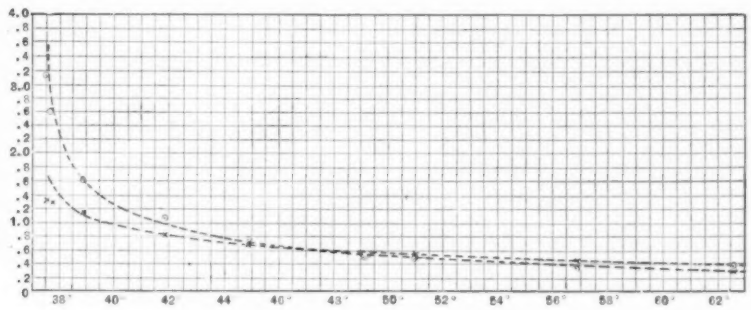


Fig. 8.

Flint-glass air flint glass, Table VII.

one is sure light can be seen, the tendency will be to underestimate rather than overestimate the diameter of the transmitted spot. Especially is this true since much more light is transmitted near the critical angle, and hence the eye rendered less sensitive to small

intensities, than is transmitted for larger angles of incidence. In the neighborhood of a degree from the critical angle the edges of the transmitted spot become fairly sharp. Furthermore, in the calculations, the value of d assumed correct is taken from a region where, for the air films, d is changing slowly. A small change in d at this point introduces, in the calculated result, a comparatively large change near the critical angle.

There appears therefore a close agreement, in every detail, both qualitatively and quantitatively, between the theory and the experimental results.

It is interesting to note that Righi[1] has measured the penetration in the case of electric waves using paraffine prisms. Unfortunately only one result is given, that for an angle of incidence of 45°, hence his results cannot be compared with theory.

THEORY FOR TWO MEDIA.

If we introduce the condition that the third medium is the same as the second we have $\cos\theta = \cos\psi$ and $n_2 = 1$. Introducing these values in equation (6) and multiplying by the conjugate imaginary the equation reduces without approximation to

$$\frac{D_s^2}{E_s^2} = \frac{4\cos^2\varphi}{\epsilon^{2u}(1 - n_1^2)}. \tag{17}$$

Similarly for the case where the electric force is parallel to the plane of incidence,

$$\frac{D_p^2}{E_p^2} = \frac{4n_1^2\cos^2\varphi}{(1 - n_1^2)(\sin^2\varphi - n_1^2\cos^2\varphi)\epsilon^{2u}}. \tag{18}$$

u has the same meaning as before, d, which enters the expression u, now means the distance of any position under consideration from the boundary.

These equations give the same qualitative relations as hold for the case where the second medium is thin, and hence three media involved. The condition that $d_s = d_p$ is that

$$n_1^2 = \sin^2\varphi - n_1^2\cos^2\varphi \quad \text{or} \quad \sin^2\varphi = \frac{2n_1^2}{1 + n_1^2},$$

which is the same as before.

[1] Righi, L'Ottica delle Oscillazioni Elettriche, p. 158.

It follows from equations (17) and (18) that the planes of constant amplitude are parallel to the bounding surface, the magnitude of the amplitude decreasing rapidly as the distance from the geometrical surface increases.

To show how rapidly the intensity falls off as the distance from the boundary increases it is necessary simply to insert values for d/λ in the above equations. The following table gives the values for the ratio D_s^2/E_s^2 for different angles of incidence and for different distances from the geometrical boundary of crown glass of index of refraction 1.5151, and air as calculated from equation (18).

ϕ	d/λ								
	¼	½	1	2	3	4	5	10	20
41°25′	3.223	2.667	1.705	0.729	0.312	0.133	0.057	$.82\times10^{-3}$	$.17\times10^{-6}$
43 4	1.615	0.744	.125	.004	$.14\times10^{-3}$	$.46\times10^{-5}$	$.15\times10^{-6}$	$.15\times10^{-13}$	$.10\times10^{-28}$
48 20	.810	.209	.014	$.62\times10^{-4}$	$.28\times10^{-6}$	$.12\times10^{-8}$	$.55\times10^{-11}$	$.95\times10^{-23}$	$.29\times10^{-46}$
60	.131	.025	$.41\times10^{-4}$	$.95\times10^{-9}$	$.21\times10^{-13}$	$.51\times10^{-18}$	$.12\times10^{-24}$	$.77\times10^{-46}$	$.34\times10^{-92}$
75	.017	$.61\times10^{-5}$	$.80\times10^{-6}$	$.12\times10^{-11}$	$.24\times10^{-23}$	$.41\times10^{-28}$	$.70\times10^{-29}$	$.11\times10^{-57}$	$.24\times10^{-116}$

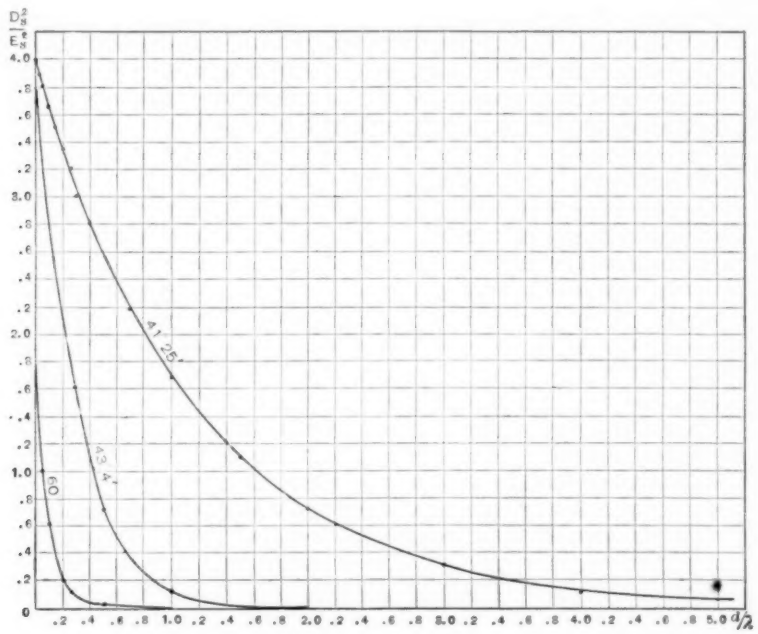


Fig. 9.

For the intensity used in the experimental work with thin films the ratio D/E was of the order of magnitude of .01 for the limit of visibility. Equation (19) would give higher values than are given in the above table for angle of incidence less than 51° .10′ (for this case of crown glass and air), and values less than the above table for angle of incidence greater than 51° 10′. Fig. 9 gives a plot of the above table.

At the critical angle where $\sin \varphi = n_1$, we have from (17) for $d = 0$, the relation $\frac{D_e}{E_e} = 2$. That this relation is true follows at once from the boundary condition derived from the principle of continuity, which is that $Y_e + Y_r = Y_2$. Both experiment and theory show that at the critical angle there is no phase difference. Since, in total reflection, the reflected energy is equal to the incident energy,[1] it follows that for an angle exceeding the critical angle by a differential amount $Y_e = Y_r$. Therefore $Y_2 = 2Y_e$.

The equation for the wave motion in the second medium for the electric force perpendicular to the plane of incidence is the real part of

$$Y_2 = D_e e^{i\frac{2\pi}{T}\left(t - \frac{x \sin \psi + z \cos \psi}{v_2}\right)}. \tag{19}$$

putting in the relation (5) equation (19) becomes

$$Y_2 = De^{i\delta} e^{-u} e^{i\frac{2\pi}{T}\left(t - \frac{x}{\frac{n_1}{\sin \phi} v_2}\right)}. \tag{20}$$

This equation represents a wave motion moving in the x direction, that is, along the boundary surface, with a velocity equal to $\frac{n_1}{\sin \varphi} \cdot v_2$. The velocity is therefore v_2 at the critical angle and decreases with an increase in the angle of incidence, reaching the minimum value of v_1 for grazing incidence. The direction of vibration of the electric force is still perpendicular to the plane of incidence and hence parallel to the boundary and transverse to the direction of propagation. The magnetic force is still perpendicular to the electric force but is not normal to the surface and hence not transverse to the direction of propagation. This must be so since a_2 is not zero.

[1] This is proved by electro-magnetic theory in Lehrbuch der Optik, p. 279.

At the boundary and for the critical angle equation (18) becomes $\frac{D_p}{E_p} = \frac{2}{n_1}$. Since n_1 is less than unity, at the critical angle the amplitude at the boundary is greater than twice the incident amplitude, and increases as n_1 decreases, that is, as the two media become more widely different. This is no contradiction to the statement that the penetration decreases as the media become more widely different for although the amplitude at the boundary may be greater, equation (18) shows that for any given angle of incidence, and given amplitude d decreases when n_1 decreases. The value $\frac{D_p}{E_p} = \frac{2}{n_1}$ for $z = 0$ at the critical angle can be derived directly from the equations of wave motion by applying the boundary condition.

Here, as in the previous case, the equation expressing the motion in the second medium may be put in a form similar to equation (20) showing the velocity of the wave along the boundary to be the same as given by equation (20). Since N_2 is not zero the amplitude is not normal to the surface, and not normal to the direction of propagation. The electric force in this case is parallel to the magnetic force in the previous case, where the electric force was perpendicular to the plane of incidence. We have then an ether wave in which the vibrations are not transverse. This is no contradiction to the Maxwell theory. The disturbance at any point is not due to the disturbance at some other point on the boundary but to the wave from the first medium incident at the point in question. This can be more clearly seen from Fig. 10. Let AB represent a particular phase of a portion of the incident wave. As the wave moves from B to O, the wave in the second medium corresponding to this particular phase will move from A to O. The motion at O is not due to the motion at A, but due directly to the incident wave from B. It can also be seen from the figure that the velocity along OA is that given by equation (20)

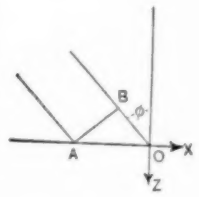


Fig. 10.

The above discussion has reference to an infinite wave, incident on an infinite plane. In the practical case, where neither the incident wave nor the bounding surface are infinite, the discussion

would be practically unchanged for points a short distance away from the edge of the incident wave or the edge of the boundary. Just what takes place at the edge is difficult to determine. It is conceivable that here a disturbance may be set up which can propagate itself along the boundary. If so Voigt's experiment for showing penetration when only two media are involved would be justified.

There seems at present no method for experimentally testing the theory for two media in the case of light waves. It would seem feasible however to test the theory with short electric waves. This the writer hopes to do at some future time.

In conclusion the writer wishes to express his appreciation of the interest shown by Professor Slate, of the University of California, and by Professors Nichols and Merritt, and Dr. Shearer, of the Physics Department of Cornell University. The writer is also grateful to Professor Gage, of the Histology Department, for the use of microscopic facilities.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
June, 1902.

ON THE EFFECT OF LOW TEMPERATURE ON THE RECOVERY OF OVERSTRAINED IRON AND STEEL.

By E. G. Coker.

THE behavior of iron and steel after overstrain has been the subject of much investigation in recent years at the hands of Ewing[1] and Muir[2] and in particular the effect of time and temperature have been very closely studied and it has been shown that iron and steel overstrained in tension when left to itself gradually recovers its elastic properties and in time the recovery is practically complete, while the yield point is raised considerably.

It has also been shown that the recovery is more rapid in wrought iron than in steel, and in either case the recovery is completed in a few minutes by exposing the material to the temperature of boiling water. It appeared, therefore, probable that exposure of an overstrained bar to a low temperature might produce a retarding effect upon the recovery. As the natural facilities afforded by a Canadian winter enable this to be readily tested the author resolved to take advantage of this and to make experiments upon the behavior of iron and steel freely exposed to the atmosphere, but shielded from temperatures above the freezing point.

The behavior of overstrained metal under low temperature is not only interesting from a purely physical point of view, but is of importance in practical applications ; thus bridge structures are liable to stresses which may cause overstrain, and the subsequent behavior of the material under the prolonged low temperatures of northern latitudes is therefore well worthy of study.

The equipment of the McGill Engineering Laboratory was available for the experiments through the kindness of Professor Bovey,

[1] Ewing, On Measurements of Small Strains in the Testing of Materials and Structures, Proc. R. S., 1895.

[2] Muir, On the Recovery of Iron from Overstrain, Phil. Trans., 1899.

and included a "Ewing" extensometer similar to others used in the researches mentioned above, and a testing machine of the Buckton type, which afforded a ready means of stressing the bars with ease and accuracy.

METHOD OF EXPERIMENT.

Previous experiments have shown that iron and steel exhibit the same tendency to recover their elasticity with time, whether the stress be tension, compression, shear or bending so that it is a matter of choice as to the way the material shall be stressed. However tension experiments are, as a rule, more easily carried out and this paper is confined to that kind of stress.

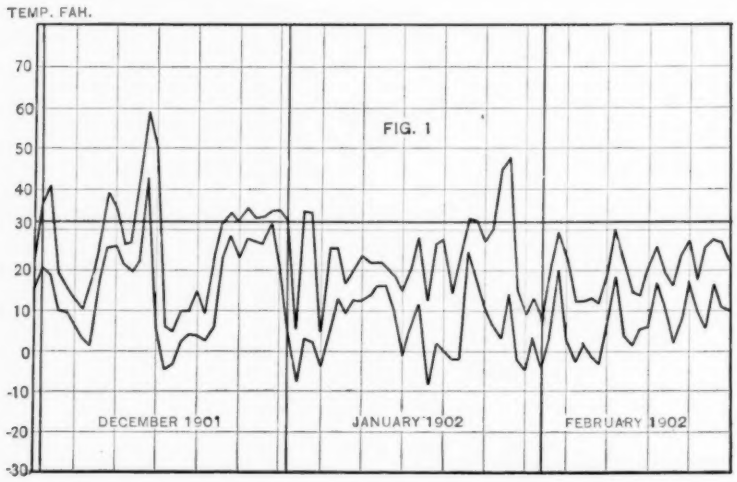


Fig. 1.

The bars under examination were first overstrained in tension and afterwards placed horizontally in the snow with a thin layer above, so that they rapidly acquire the outside temperature. This layer was constantly renewed during a thaw to prevent the temperature rising above 32° Fahr. The temperature therefore roughly corresponded to that of the atmosphere, but was not allowed to rise above the melting point of snow, except during short intervals when the condition of the specimen was examined under stress in the testing machine.

In order to show the variations of temperature while the experiments lasted, the maximum and minimum temperatures recorded in the University Observatory, close by, are plotted in Fig. 1, from November 30, 1901, the beginning of the experiments, until February 23, 1902, when they concluded. Soon after this date there was a prolonged thaw and the snow disappeared so rapidly that it was found impossible to keep the bars covered by snow, and the experiments were therefore discontinued. An inspection of the temperature diagram shows that while the work was in progress the temperature was below freezing the greater part of the time and no difficulty was experienced in keeping the bars at or below the meeting point of ice.

The Effect of Low Temperature on Steel Bars.

Two annealed bars were taken, cut from a piece of boiler plate, and these were stressed beyond the yield point in the usual way, and immediately after the condition of each bar was examined by resetting the extensometer and subjecting the material to a cycle of stress starting from no load and rising to an upper limit, in close proximity to the initial yield point and afterwards returning to zero. The stress-strain curves for this second loading showed the well-known hysteresis effect, and afforded a standard by which to compare the behavior of the bars on subsequent occasions. If a bar overstrained in this way be allowed to rest in a room at ordinary temperature, it will begin to recover at first very rapidly, the rate of recovery diminishing as time goes on, but in general after the lapse of three or four weeks a steel bar regains its primitive elastic condition, provided the permanent extension is not more than two or three per cent.; it has, however, a new yield point much higher than before. As a means of comparison with the two bars under test; some previous results on a bar cut from the same plate may be quoted. This bar had a breadth of 2.07 inches, a thickness of 0.377 inch and a length under test of eight inches. It was stressed until a permanent set of .165 inch was obtained and the following observations were made, the unit of extension throughout being $\frac{1}{10000}$ inch.

Load Pounds.	Reading.	Difference.
0	200	
		70
4,000	270	
		69
8,000	339	
		69
12,000	408	
		71
16,000	479	
		74
20,000	553	
23,000	went off scale.	
27,000	Total Length 8.165 inches.	

The condition of the bar was now examined by subjecting it to a cyclical loading rising to 24,000 pounds with the following result:

Load Pounds.	Reading.	Difference.
0	200	
		69
4,000	269	
		74
8,000	343	
		79
12,000	422	
		82
16,000	504	
		94
20,000	598	
		99
24,000	697	
		70
20,000	627	
		79
16,000	548	
		79
12,000	469	
		81
8,000	388	
		84
4,000	304	
		89
0	215	

These relations between stress and strain are plotted in Fig. 2, and are marked I. and II. respectively.

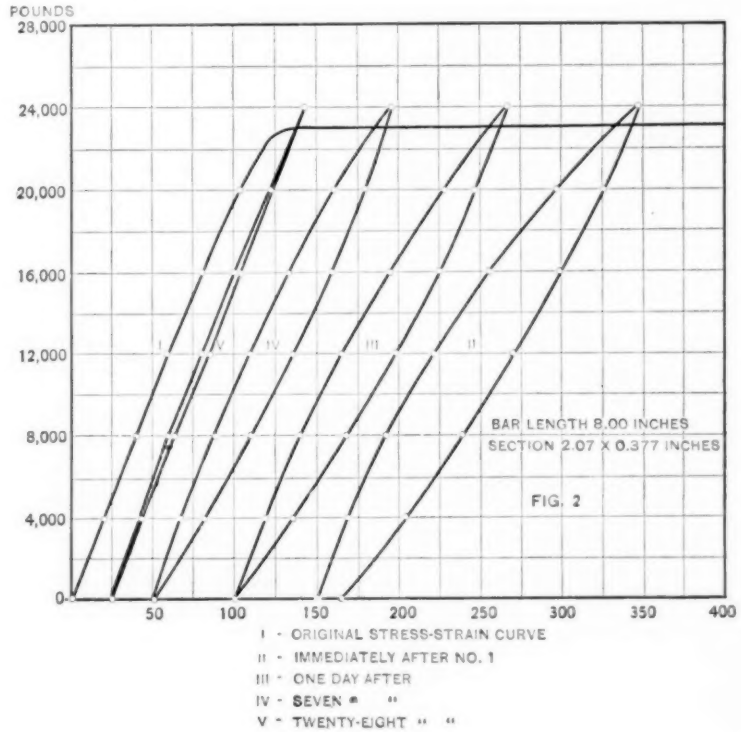
The figures representing the actual strains are, however, reduced by subtracting therefrom twenty-five units for each 2,000 pounds of load, thereby shearing back the curves toward the vertical and exhibiting the characteristic differences in a striking manner. This convenient method of plotting, due to Professor Ewing,[1] is used throughout this paper.

The cyclical loading was repeated at the end of three, seven and twenty-eight days and the observations are recorded on Table I., and plotted on the accompanying Fig. 2.

[1] Muir, On the Recovery of Iron from Overstrain, p. 12.

TABLE I.

Load in Pounds.	One Day After.		Seven Days After.		Twenty-eight Days After.	
	Reading.	Δ	Reading.	Δ	Reading.	Δ
0	200		200		200	
		−69		−68		−67
4,000	269		268		267	
		−70		−70		−68
8,000	339		338		335	
		−75		−71		−70
12,000	414		409		405	
		−81		−75		−70
16,000	495		484		475	
		−84		−77		−72
20,000	579		561		547	
		−88		−86		−71
24,000	667		647		618	
		−70		−68		−68
20,000	597		579		550	
		−73		−70		−70
16,000	524		509		480	
		−76		−74		−69
12,000	448		435		411	
		−82		−77		−71
8,000	366		358		340	
		−83		−77		−70
4,000	283		281		270	
		−85		−79		−71
0	198		2		199	



I - ORIGINAL STRESS-STRAIN CURVE
II - IMMEDIATELY AFTER NO. 1
III - ONE DAY AFTER
IV - SEVEN " "
V - TWENTY-EIGHT " "

Fig. 2.

The diagrams show at first a rapid recovery, diminishing with the time, but practically complete in four weeks and are characteristic of the material under the conditions.

We may now compare the behavior of this specimen with a similar specimen kept at the outside temperature. The bar was 2.06 inches in width, .405 inch thick and had a length under test of eight inches, and the unit reading of the extensometer remained the

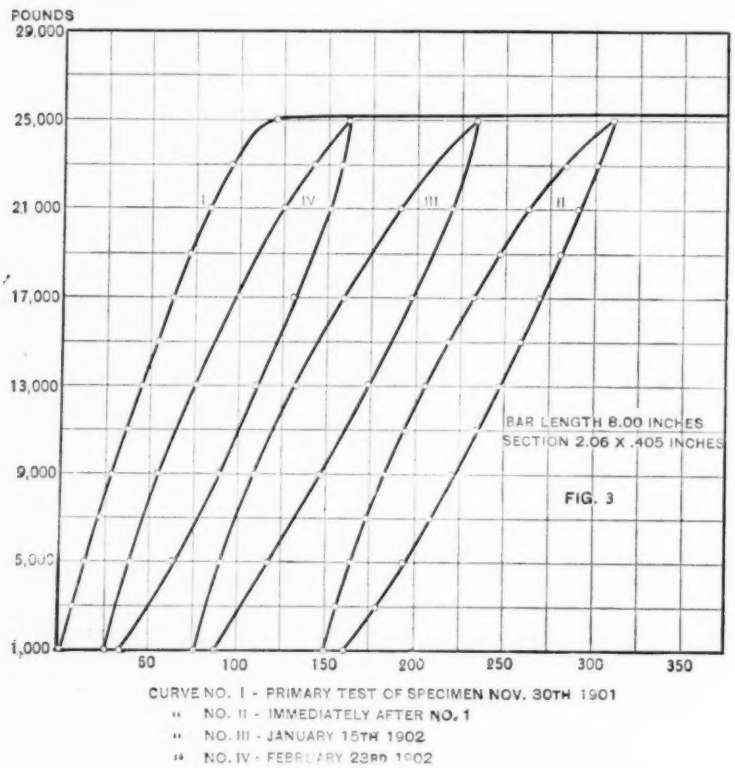


Fig. 3.

same. The primitive stress strain curve and the hysteresis loop, determined immediately after, are shown in Fig. 3, and the readings are recorded in Table II., from which it will be seen that the agreement is comparatively close; after about six weeks' exposure the stress cycle was repeated and found to be practically unchanged, showing the almost complete arrest of the recovery.

TABLE II.

1 *division* = $\frac{1}{30000}$ *inch. Bar: Length* = 8.00 *inches. Section* = 2.06 × 4.05 *inch.*

Load in Pounds.	November 30, 1901.		Immediately After.		January 15, 1902.		February 23, 1902.	
	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ
1,000	200		200		200		200	
		−64		− 65		−64		−66
5,000	264		265		264		266	
		−67		− 70		−71		−65
9,000	331		335		335		331	
		−65		− 72		−73		−71
13,000	396		407		408		402	
		−67		− 76		−76		−74
17,000	463		483		484		476	
		−71		− 80		−81		−75
21,000	534		563		565		551	
		−86		−100		−91		−92
25,000	620		663		656		643	
				− 71		−62		−65
21,000			592		594		578	
				− 71		−72		−75
17,000			521		522		503	
				− 73		−74		−69
13,000			448		448		434	
				− 77		−75		−70
9,000			371		373		364	
				− 80		−81		−76
5,000			291		292		288	
				− 81		−81		−79
1,000			210		211		209	

The bar was now put back for another five weeks and after the expiration of that time again subjected to test. The observations,

TABLE III.

1 *division* = $\frac{1}{30000}$ *inch. Bar: Length* = 8.00 *inches. Section* = 2.06 × .374 *inches.*

Load in Pounds.	November 30, 1901.				January 15, 1902.		February 23, 1902.	
	Primary Overdrain.		Immediately After.					
	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ
1,000	200		200		200		200	
		−71		−72		− 72		−72
5,000	271		272		272		272	
		−71		−77		− 76		−75
9,000	342		349		348		347	
		−73		−80		− 80		−79
13,000	415		429		428		426	
		−72		−83		− 83		−84
17,000	487		512		511		510	
		−73		−86		− 89		−85
21,000	560		598		600		595	
		−73		−91		− 87		−89
25,000	633		689		687		684	
		−78		−98		−102		−94
29,000	711		787		789		778	
		−68		−53		− 53		−47
31,000	779		840		842		825	
				−36		− 37		−34
29,000			804		805		791	
				−76		− 80		−71
25,000			728		725		720	
				−77		− 76		−77
21,000			601		649		643	
				−81		− 78		−79
17,000			570		571		564	
				−85		− 87		−82
13,000			485		484		482	
				−86		− 84		−82
9,000			399		400		400	
				−88		− 86		−87
5,000			311		314		313	
				−93		− 94		−94
1,000			218		220		219	

Table II., showed some recovery had taken place, but the bar was still very far from its original condition. Owing to a long continued thaw soon after this, it was found impossible to keep the bar covered by snow and the experiment came to an end. These results are confirmed by the behavior of the companion test piece (Fig. 4) which was subjected to exactly similar treatment. The fig-

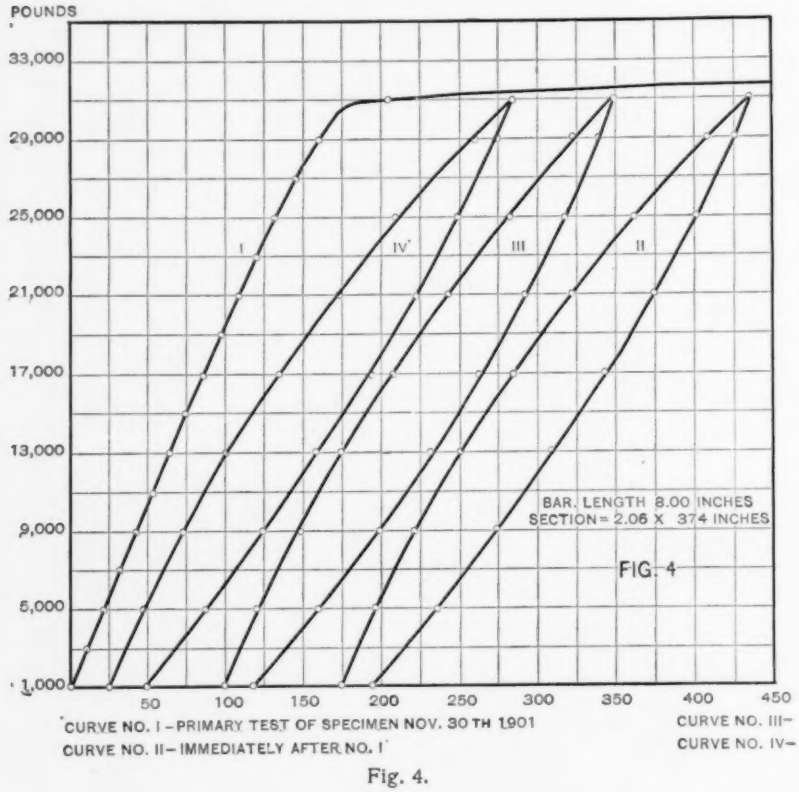


Fig. 4.

ures on Table III. show that there was no recovery during the first six weeks, and at the end of five weeks more some recovery was noticeable. It is, therefore, apparent that a temperature below that of melting ice will cause a very complete arrest of the recovery of an overstrained steel bar, its effect being the exactly opposite to that of temperature at or near the boiling point of water, which latter causes an immediate recovery of the primitive elastic properties.

The Effect of Moderate Low Temperature on Overstrained Wrought-Iron Bars.

It has been pointed out that iron recovers from overstrain at a comparatively rapid rate as compared with steel, and its behavior under low temperature may therefore be somewhat different from steel.

As the conditions of experiment were very nearly the same in both cases, the results afford a comparison of the behavior of iron and steel under corresponding conditions.

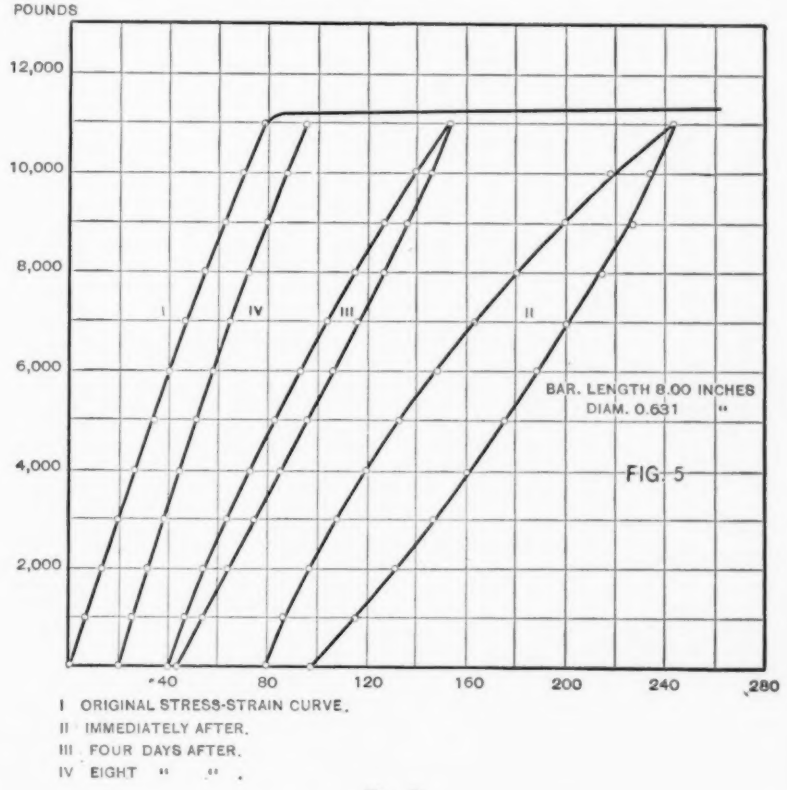


Fig. 5.

Of the three bars chosen, cut from the same rod, one was reserved for a test at the temperature of the laboratory, which never differed much from 65° Fahr., the other two were overstrained and placed alongside the steel bars in the snow. The rapid recovery of iron in the case of the first specimen is well shown by

the observations which are given in Table IV., and are represented graphically in Fig. 5.

TABLE IV.

1 *division* = $\frac{1}{50000}$ *inch.* *Bar: Length* = 8.00 *inches.* *Diam.* = 0.631 *inch.*

Load in Pounds.	December 20, 1901.				December 24, 1901.		December 28, 1901.	
	Primitive Loading.		Immediately After.					
	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ
0	200	−93	200	− 93	200	− 94	200	−92
2,000	293	−93	293	−107	294	−100	292	−93
4,000	386	−94	400	−108	394	−100	385	−92
6,000	480	−94	508	−112	494	−101	477	−94
8,000	574	−92	620	−117	595	−104	571	−98
10,000	666	−52	737	− 65	699	− 55	669	−50
11,000	718		802	− 49	754	− 49	719	−48
10,000	12,500 lbs.=8.3″		753	− 99	705	− 98	671	−98
8,000			654	−106	607	−101	573	−94
6,000			548	−108	506	−100	478	−94
4,000			440	−108	406	−102	385	−94
2,000			332	−114	304	−101	291	−92
0			218		203		199	

The recovery in the case of this bar is nearly four times as rapid as the corresponding steel bar. It should, however, be noted that through the accidental circumstance of the permanent stretch 0.3 inch being more than was intended, the rate of recovery is not strictly comparable, but it shows sufficiently well the difference in the behavior of the two.

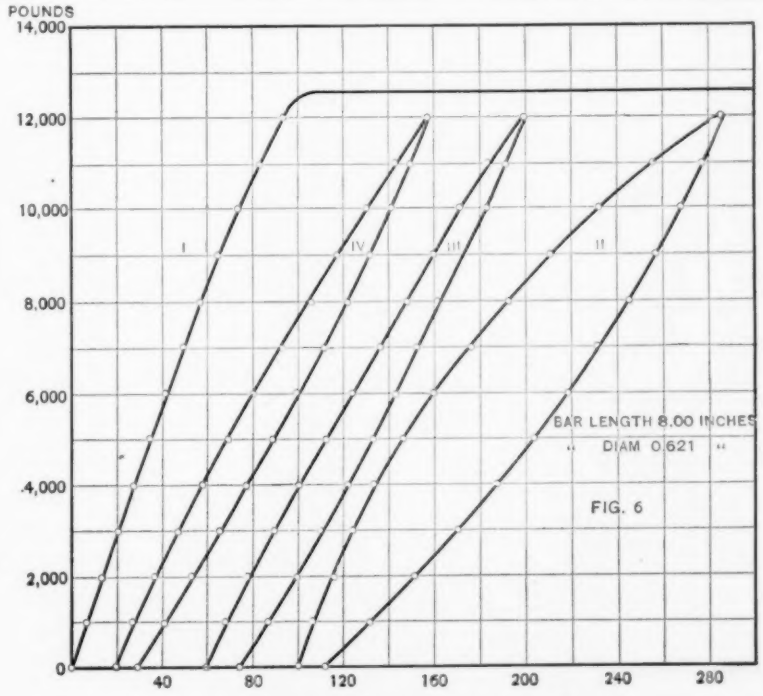
As the dimensions and loads were so different from the previous cases it was found convenient in plotting the observations for the wrought-iron specimens to shear back the curves in the ratio of forty units for each thousand pounds of load.

The recovery of a similar bar when subjected to moderate low temperature is, as might be expected, quite different from the former case, but it does not resemble steel in its behavior. This is illustrated by the case of a wrought-iron specimen, which immediately after overstrain on December 7 was placed in the snow and allowed to remain there until January 15 following. The observations, Table V., showed that a decided recovery had taken place during the interval. The bar was put back and again tested on February 23, and showed only a slight recovery.

TABLE V.

Wrought-iron bar C. Mean diameter = 0.621 *inch. Length* = 8.00 *inches.*
1 *division* = $\frac{1}{30000}$ *inch.*

Load in Pounds.	December 7, 1901. Primitive Loading. Reading.	Δ	December 7, 1901. Immediately After. Reading.	Δ	January 15, 1902. Reading.	Δ	February 23, 1902. Reading.	Δ
0	200	−95	200	− 94	200	− 98	200	− 96
2,000	295	−94	294	− 97	298	−103	296	−103
4,000	389	−93	394	−107	401	−103	399	−102
6,000	482	−95	501	−113	504	−106	501	−107
8,000	577	−98	614	−118	610	−102	608	−101
10,000	675	−97	732	−136	712	−109	709	−108
12,000	772		868	− 99	821	− 97	817	− 96
10,000	12,000 lbs. = 82		769	−102	724	−102	721	−100
8,000			667	−107	622	− 98	621	−102
6,000			560	−113	524	−101	519	−102
4,000			447	−115	423	−104	417	−104
2,000			332	−122	319	−106	313	−103
0			210		213		210	



I - ORIGINAL STRESS-STRAIN CURVE, DEC. 7TH 1901.
II - IMMEDIATELY AFTER NO. 1.
III - JANUARY 15TH 1902
IV - FEBRUARY 23RD 1902

Fig. 6.

These observations are plotted in Fig. 6 in which II. is the hysteresis loop immediately after overstrain, which, as before, serves as a standard of comparison of the behavior of the bar.

The recovery at the end of the first period is apparent by the large diminution in the hystereses between stress and strain, but there was practically no further recovery after a second interval of about the same duration as is shown by the almost precisely equal loop marked IV.

TABLE VI.

Wrought-iron bar D. Mean diameter = 0.637. *Length* = 8.00 *inches.*
1 *division* = $\frac{1}{30000}$ *inch.*

Load in Pounds.	Primitive Loading.		Immediately After.		January 15, 1902.		February 23, 1902.	
	Reading.	Δ	Reading.	Δ	Reading.	Δ	Reading.	Δ
0	200	−93	200	− 98	200	− 93	200	− 94
2,000	293	−91	298	− 99	293	− 99	294	−100
4,000	384	−93	397	−107	392	− 97	394	−101
6,000	477	−91	504	−110	489	−102	495	−102
8,000	568	−91	614	−119	591	−102	597	−103
10,000	659	−90	733	−137	693	−118	700	−103
12,000	749		870	−100	811	−108	803	− 93
10,000	12,500 lbs. = 8		770	−101	703	− 97	710	− 99
8,000			669	−108	606	− 97	611	− 98
6,000			501	−112	509	−101	513	−101
4,000			449	−115	408	−102	412	−101
2,000			334	−120	306	−104	311	−105
0			214		202		206	

The second bar under precisely similar conditions, except that the permanent stretch was greater, gave similar results, as is shown by Table VI., in which the diminution of the overstrain during the first period is large but is practically reduced to nothing in the second interval.

This anomalous behavior of wrought iron after overstrain may be explained if we consider that beyond a certain point the action of the low temperature is insufficient to maintain the condition of overstrain, which therefore gradually diminishes until a point is reached at which the tendency to recover is balanced by the opposing action of temperature.

On this view the tendency to recover from overstrain in the case of the steel is entirely balanced by the opposing effect of the temperature and its recovery is in consequence entirely arrested.

LECTURE ROOM DEMONSTRATIONS OF ASTIGMATISM AND OF DISTORTION.

By W. S. Franklin.

THE astigmatism of a lens for an oblique pencil of rays may be shown subjectively by looking obliquely through a magnifying glass at the rulings on a piece of cross-section paper. The lens may be adjusted to show one set of rulings distinctly, when, upon moving the lens nearer to or farther from the paper, the other set of rulings will become distinct. The conditions necessary to bring out the astigmatism in a striking manner are that a narrow pencil of rays be employed passing through a small portion of the lens, and that the field angle be narrow. This is substantially realized in the above use of the magnifying glass inasmuch as the pencil of rays from any given point in the sheet of paper is delimited by the pupil of the observer's eye, and inasmuch as the field angle where the eye is fixed upon a given point of the paper is determined by the extent of the region of distinct vision, that is by the size of the yellow spot in the retina of the eye.

The astigmatism of a lens for an oblique pencil of rays may be shown objectively by projecting on the screen an image of a cross-ruled transparent plate, the projecting lens being turned so that its axis makes an angle of twenty or thirty degrees with the optical axis of the lantern. Under these conditions the vertical or horizontal lines may be sharply focused at will and if the lines are very fine it may easily be that the one set of lines is rendered invisible by blurring when the other set of lines is sharply in focus. A small portion only of the plate should be used, the remainder being cut off by a diaphragm; and the lantern should be adjusted so that the light from the condenser may be focused as it passes through the projecting lens.

The error of a lens known as distortion, an error which is exhibited to a marked degree by the so-called view lens of the ama-

teur photographer, may be demonstrated by means of the projection lantern and the principle of rectilinear lens may be shown. In this connection it should be remembered that the distortion produced by a simple lens or by a simple achromatic doublet is very small except when a stop or diaphragm is used at some distance in front of the lens to improve the definition. Instead of a diaphragm it is more convenient to adjust the lamp in the lantern so that the condenser lenses focus the light in front of or behind the projecting lens as may be desired, the position of this focus corresponding to the position of a diaphragm or stop. To show this error as large a field angle as possible must be used.

A large cross-ruled transparent plate is placed in the lantern and projected by means of a simple biconvex lens. If the lamp is adjusted until the focus of the condensing lenses lies, approximately, in the plane of the projecting lens and the projecting lens is adjusted to give the sharpest possible image on the screen, the image shows little or no distortion. If the lamp is adjusted as that the condensing lenses focus the light some distance in front of the projecting lens, and the projecting lens is adjusted to give the sharpest possible image on the screen, the image is distorted, the lines being concave on the side towards the center of the field. If the lamp is adjusted so that the condensing lenses focus the light some distance beyond the projecting lens and the projecting lens is adjusted to give the sharpest possible image on the screen, the image is distorted, the lines being convex on the side towards the center of the field.

In the symmetrical orthoscopic or rectilinear doublet the diaphragm or stop is midway between the lenses of the doublet, that is in front of one lens and behind the other, and the distortion of one lens is opposite to that of the other. The distortion is thus compensated.

PRISMS AND PLATES FOR SHOWING DICHROMATISM.

By R. W. Wood.

THE usual method of showing dichromatism, or the change in the tint of the transmitted light, with increasing thickness of the absorbing medium, is to superpose two or more stained films or strips of suitably tinted glass. The method has a disadvantage in that it does not demonstrate very clearly the cause of the change of color. As is well known the change is due to the fact that the coefficient of absorption is greater for one absorbed region than for some other. If the medium be given the form of an acute prism, and a source of light be viewed through it, the transmitted colors are separated, and the change in the intensities of each can be followed as the thickness increases by moving the eye from the refracting edge towards the base.

I have made prisms of this nature in the following way: A quantity of Canada balsam is boiled down until it will harden on cooling, and a quantity of brilliant green is dissolved in it. The dye must not be added to the balsam until it has cooled almost to the point of becoming thick, otherwise it will be decomposed and a very muddy green result. The amount of the dye should be such that a layer of the liquid 1.5 cm. thick appears blood red. Thin layers appear blue, and if a prism be made of this mixture it will show a blue-red dichromatism. A green-red dichromatism is much more striking and the requisite green color can be produced by the addition of a quantity of naphthol yellow. This makes a beautiful dichromatic medium. A small hollow prism made of two strips of plate glass mounted together with wooden sides, in which grooves have been cut, is warmed and filled with the mixture. As soon as the balsam is hard a small quantity of melted sealing wax is poured on top of it which materially increases the strength of the prism. A lamp flame viewed through this prism is separated into a red image and a green image, and as the eye is moved along towards the base the

green image fades rapidly away, while the red one scarcely changes at all.

If a broad flame be used the overlapping portions of the two images will be seen to be colored yellow, secondary yellow due to the mixture of red and green light on the retina. The thickness of the prism at the point where this yellow is purest is the critical thickness, or the thickness at which the change from red to green takes place. If we consider the color change as a subjective phenomenon there are three distinct colors seen with different thicknesses of this medium, green, yellow and red. This can be shown better by making the balsam solution of the dye much stronger and pressing it out between plates of glass. It is easy to prepare in this way plates very slightly prismatic which are red along one side, yellow in the middle, and green along the other side. The yellow is always a little puzzling especially if a flame is the source of light looked at. The lack of acromatism in the eye causes the yellow image to have red or green borders, the effect being difficult to describe. In the case of one of my plates the color of the transmitted light is almost exactly that of the sodium flame, and yet a sodium flame is absolutely invisible through it.

The dependence of the color on the original composition of the light is also well shown by these plates, some of them appearing red by lamplight and bright green by daylight, a phenomenon exhibited by the well-known Alexandrite crystals.

I have a prism which I made several years ago which is dichromatic by lamplight, but not by daylight, which illustrates well that dichromatism depends on the relative intensities of the different parts of the spectrum of the light used, as well as on the different absorption coefficients. If I remember rightly this prism was made of malachite green.

The prisms should be made about 4 cm. long, and 2 cm. thick at the base.

THE INVISIBILITY OF TRANSPARENT OBJECTS.

By R. W. Wood.

A TRANSPARENT body, no matter what its shape, disappears when immersed in a medium of the same refractive index and dispersion. Could a transparent solid substance be found, whose refractive index and dispersion were the same as those of air, it would be absolutely invisible. We can find a solid, the refractive index of which is equal to that of air for light of a certain wave-length, but unfortunately it is not a transparent substance. If it were we should have an example of a solid absolutely invisible when illuminated with monochromatic light. The dispersion curves of some of the aniline dyes cross the line of unit refractive index for certain wave-lengths, but the point of crossing is in the immediate vicinity of the absorption band. Cases of this nature will be discussed more fully under another heading.

The disappearance of a transparent substance when immersed in a medium of identical optical properties is usually illustrated by dipping a glass rod into Canada balsam, but the disappearance is not complete, for the dispersion of the glass and the liquid are not the same. A better fluid is a solution of chloral hydrate in glycerine which is quite colorless. The glycerine is to be heated and the crystals added until the refractive index is of the required value. The hot liquid will dissolve a most astonishing amount of the chloral, ten or fifteen times its bulk, if I remember right, and only a very small amount should be placed in the beaker to start with. This solution has almost exactly the same dispersion as glass, and finely powdered glass stirred up in it becomes perfectly transparent without showing a trace of Christiansen's colors. A glass rod disappears completely when dipped into it and when withdrawn presents a curious aspect, for the end appears to melt and run freely in drops.

Lord Rayleigh in his article on optics in the *Encyclopædia Brittanica* points out that perfectly transparent objects are only visible

in virtue of non-uniform illumination, and that in uniform illumination they would become absolutely invisible. A condition approaching uniform illumination might, he says, be attained on a top of a monument in a dense fog. I have recently devised a method by which uniform illumination can be very easily obtained and the disappearance of transparent objects when illuminated by it illustrated. The method in brief is to place the object within a hollow globe, the interior surface of which is painted with Balmain's luminous paint and view the interior through a small hole.

The apparatus can be made in a few minutes in the following manner: A quantity of Canada balsam is boiled down, until a drop placed on cold glass solidifies. The Balmain paint, in the form of a dry powder is stirred into the hot balsam until the whole has the consistency of thick paint. Two glass evaporating dishes of equal size are carefully cleaned and warmed and coated on the outside with the hot mixture, which can be flowed over the glass, and by the dexterous manipulation of a small Bunsen flame made to cover the entire outer surface. I first tried painting the inside of a copper sphere with the commercial paint, but the surface was unsatisfactory and could not be easily cleaned. Probably two perfectly plain hemispherical finger-bowls could be used instead of the evaporating dishes. As soon as the coating has become hard a small hole is cut through it to enable the interior to be viewed. If the lips of the dishes are placed together the interior can be seen through the small opening, but in this case the line of junction, which is always more or less dark, comes opposite the aperture, which is a disadvantageous arrangement.

If the inner surfaces be exposed to bright daylight, sun or electric light, and the apparatus taken into a dark room, a crystal ball or the cut glass stopper of a decanter placed inside, it will be found to be quite invisible when viewed through the small aperture. A uniform blue glow fills the interior of the ball and only the most careful scrutiny reveals the presence of a solid object within it. One or two of the side facets of the stopper may appear if they happen to reflect or show by refraction any portion of the line of junction of the two hemispheres.

A METHOD FOR MAINTAINING INTERMEDIATE TEMPERATURES.

By Chas. T. Knipp.

LAST August, while working on the surface tension of liquid air,[1] a number of experiments were tried using the liquid as a refrigerant. Of these one was thought to be of general interest, and believing that it is new, since nothing has come to my notice of a similar method, I feel warranted in submitting this note.

At the time measurements on the resistance of manganin wire at liquid air temperatures were being conducted. The terminals were of No. 2 bare copper wire and about 30 cm. long. These were dipped into mercury cups in the usual manner. The ends to which the manganin wire was fastened were dipped into liquid air contained in an ordinary Dewar bulb to a depth of about 6 centimeters. No attempt was made to prevent radiation. At the end of ten or fifteen minutes I was surprised to find the mercury in the cups frozen. This at once suggested a possible means of maintaining intermediate temperatures. The following apparatus was constructed.

A coil of two layers containing about six turns each was made of the same copper wire. The ends of the wire were left about 25 cm. long. The coil which was compactly wound and heavily wrapped in an asbestos jacket *MN*, was mounted over a Dewar bulb as shown in Fig. 1. The distance of the coil from the upper surface of the liquid was about 20 cm. A glass tube containing about a cubic centimeter of mercury was inserted in the coil. The following data show very strikingly how quickly the heat was abstracted from the coil.

	A. M.
Mounted apparatus on bulb at	10:45
Inserted tube containing mercury,	11:00
Mercury began to freeze,	11:04
Mercury all frozen,	11:06¼

[1] Phys. Rev., Vol. XIV., No. 2, February, 1902.

At first it was doubted whether fifteen minutes was a sufficient length of time to cool the coil to a constant temperature. However, upon heating the mercury to room temperature and again inserting it in the coil it was found to freeze within the above time, thus showing that the constant temperature was reached in a very short time. Measurements on the temperature of the solidified mercury and also with different forms of apparatus should have been made, but limited time prevented.

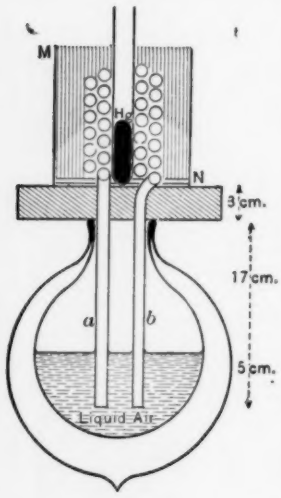


Fig. 1.

By making the conducting rod (for one rod will answer equally well) large and replacing the coil at the top by a block of copper and the whole carefully jacketed by a non-conducting material, we can make the steady temperature which the apparatus acquires approach as near to the temperature of the refrigerant as we please. It is evident that in order to maintain a constant temperature, the radiation conditions, also (to a less extent) the liquid level of the refrigerant in the bulb, must be kept constant. Again, by increasing the length of the conducting material or decreasing its area or both, constant intermediate temperatures can be maintained. The apparatus can be made in most any form to meet the needs of the experimenter.

The constant conduction of heat into the refrigerating liquid makes it evaporate rapidly, requiring that the liquid be renewed from time to time. This indicates that the method is not economical. However, the advantages are obvious. It is safe to assert that one gallon of liquid air can be made to maintain any intermediate temperature for a period of several hours. After all, the cost is a small item when compared to the advantages accruing to the investigator.

NEW BOOKS.

Vector Analysis. By Edwin Bidwell Wilson, Ph.D. New York, Charles Scribner's Sons; London, Edward Arnold, 1901. 8vo, pp. xviii + 436.

This volume is an adaptation of the lectures of Professor Gibbs, with some use of the works of Heaviside, Föppl, and Hamilton. It appears as one of the YALE BICENTENNIAL PUBLICATIONS.

Chapter I. develops the usual analysis of addition, subtraction, and scalar multiplication of vectors, with applications to geometry and physics. A bold face type designates a vector, and usually, the corresponding italic type designates the magnitude of the vector.

In Chapter II., so-called products of vectors are defined :

$$\text{the } \textit{direct product}, \ \mathbf{a} \cdot \mathbf{b} = ab \cos \angle (\mathbf{a}, \mathbf{b}) ;$$

$$\text{the } \textit{skew product}, \ \mathbf{a} \times \mathbf{b} = \text{that vector which}$$

represents the area of the parallelogram $(\mathbf{a}, \mathbf{b})$ in magnitude and axis, viz., its magnitude is

$$ab \sin \angle (\mathbf{a}, \mathbf{b})$$

and it is perpendicular to the plane $(\mathbf{a}, \mathbf{b})$ on that side from which the turn from $\mathbf{a}$ to $\mathbf{b}$ appears counter clockwise.

These products give rise to others, such as

$$[\mathbf{abc}] = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \text{volume of parallelopiped } (\mathbf{a}, \mathbf{bc}), \text{ etc.}$$

With this addition, applications to geometry, and the mechanics of a rigid body, are made.

Chapters III. and IV. treat of differentiation and integration. Hamilton's ∇, which gives the direction and rate of most rapid increase of a scalar function of the position of a variable point, is also introduced. Mr. Wilson proposes the name of *del* for this symbol, not knowing the fact that *nabla* was adopted at a gathering of leading physicists of England, Canada and the United States, in 1884. Lord Kelvin's suggestion, and the final result, appear in my report of his lectures, pp. 78, 112.

The questions of line, surface and volume integrals and their relations are very thoroughly considered. A number of differential and integral operations of physical importance are named. Thus

$$\nabla \ = \text{div} - \text{divergence of}$$

$$\nabla \times = \text{curl} = \text{curl of}$$

$$\iiint \frac{V_2}{r_{12}} dv_2 = \text{Pot } V_1.$$

Also Laplacian, Maxwellian, and Newtonian integral operations are denoted by *Lap.*, *Max.*, *New.* The mutual potential

$$Pot(U, V) = \iiint\iiint \frac{U_1 V_2 dv_1 dv_2}{r_{12}}, \quad \text{etc.}$$

Chapter V. considers Hamilton's theory of linear functions, with the symbolic duads $\mathbf{ab}\cdot$, $\mathbf{ab}\times$, as base symbols. The two concluding chapters consider rotations and strains, applications to the propagation of light in crystals, harmonic vibration, etc.

A most valuable feature is the synopsis of formulas and results which appears at the end of each chapter.

The volume is an effort to develop a vector analysis, and while valuable, it can be neither as extensive nor as powerful as the already complete quaternion analysis. In fact it becomes quaternion analysis of a very limited form by the changes

$$\mathbf{a}\cdot\mathbf{b} = S\cdot\mathbf{ab} = \text{scale of } \mathbf{ab} = -\text{ scalar of } \mathbf{ab},$$

$$\mathbf{a}\times\mathbf{b} = V\cdot\mathbf{ab} = \text{vector of } \mathbf{ab},$$

$$\mathbf{ab}\cdot = \mathbf{a}S\cdot\mathbf{b}, \quad \mathbf{ab}\times = \mathbf{a}V\cdot\mathbf{b}.$$

I do not understand why vector writers are not satisfied with the quaternion functional forms, $S\cdot\mathbf{ab}$, $V\cdot\mathbf{ab}$, which get one very soon into a practically useful analysis, without inventing any multiplication system at all. Due regard for the previously established notation and its large number of users would suggest the propriety of doing this, especially when the changed forms are of doubtful improvement.

In the first place, symbolic letters are as good as marks for denoting operations, when once accustomed to them ; witness Newton's *dot* versus Leibnitz's *d*, for differentiation. Again, the product forms obtained by the marks are confusingly irregular in their laws, being variously non-commutative, non-associative and non-distributive, while division is indeterminate. Quaternions is *the only linear algebra* after ordinary algebra, in which multiplication is always associative and distributive, and division always determinate ; and one can imagine the interest it would create had Gibbs' analysis been first in the field, and it was discovered that there was such a multiplication system underlying it, which both simplified and generalized that analysis.

ARTHUR S. HATHAWAY.

ROSE POLYTECHNIC INSTIUTTE.

ELECTRICALLY-DRIVEN TUNING FORK

The tuning fork is ten inches long and very heavy. The vibrating device is so arranged that there is practically no buzzing and the note from the fork is full and clear. Weights to be attached to the prongs of the fork are provided so that any desired pitch can be obtained up to middle C.

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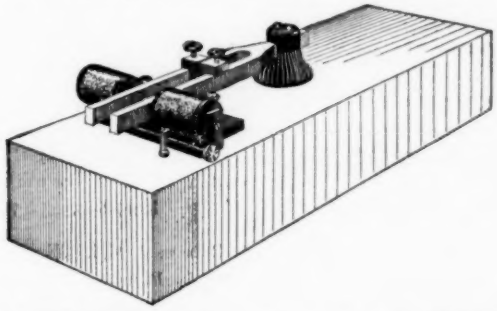

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